# Particle Filters for Non-Gaussian Hunt-Crossley Model of Environment in Bilateral Teleoperation

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Abstract—Optimal solution for nonlinear identification problem in the presence of non-Gaussian distribution measurement and process noises is generally not analytically tractable. Particle filters, known as sequential Monte Carlo method (SMC), is a suboptimal solution of recursive Bayesian approach which can provide robust unbiased estimation of nonlinear non-Gaussian problem with desire precision. On the other hand, Hunt-Crossley is a widespread nonlinear model for modeling telesurgeries environment. Hence, in this paper, particle filter is proposed to capture most of the nonlinearities in telesergerie environment model. An online Bayesian framework with conventional Monte Carlo method is employed to filter and predict position and force signals of environment at slave side respectively to achieve transparent and stable bilateral teleoperation simultaneously. Simulation results illustrate effectiveness of the algorithm by comparing the estimation and tracking errors of sampling importance resampling (SIR) with extended Kalman filter.

Keywords—Nonlinear Identification, Gaussian Noise, Force Predictions, Hunt-Crossley, Telesurgery, Particle Filter.

# I. INTRODUCTION

Teleoperation systems typically consists of a master robot for operator's leverage and providing force feedback, a slave robot which imitate master commands and interacts with the environment, and a communication channel for transition of data. In bilateral teleoperation systems an operator uses robotic manipulator to accomplish tasks at a distance, while force feedback from remote environment is resubmit to recover the transparency [1]. Transparency is a performance objective in teleoperated system which compares the amount of distortion employ by the teleoperation system while accomplishing a task with the direct execution of it. In other words, an ideal transparent teleoperator system dose not introduce any dynamics relating to system's own characteristics during execution of tasks [2].

Force reflecting teleoperation has been cited extensively in the recent research area [2]. One of the main concerns in the design of the force-reflecting teleoperators is the enhancement of transparency while assuring stability of the global system under widest possible uncertainty bounds persist in communication channel as well as environmental dynamics. In these systems, the stability concern arises due to the structure of the closed loop system. The strong force feedback creates a destabilizing effect known as induced master motion phenomenon [3]. Despite the enormous amounts of literature dealing with the design of guaranteed stable feedback systems, they present undesirable poor transparency. This constitutes the major challenge of designing the force-reflecting teleoperators.

Motion filtering and force prediction of robotic manipulators is inevitable from two different points of view. First, sensors are polluted by measurement noise while noise-free signals are required in the control structure. Secondly, force sensors are inaccurate, hard to use, and expensive [4]. Therefore, online identification methods are required to smooth and estimate desire signals. Different environment identification methods have been proposed in literature for linear and Gaussian noise in teleoperation systems (e.g. [5]–[7]). However, in the case of nonlinearity like in Hunt-Crossley model with non-Gaussian process noise, there is no optimal solution for the tracking problems. Although, different methods have been proposed for identification of Hunt-Crossley environment model like in [8], [9], establishing a probability framework for robust identification of environment dynamics has not been reported.

One of the main advantages of probability framework is that it provides the opportunity of decision making under uncertainty which is a unique feature among other approaches. Besides, knowledge fusion is significantly facilitated in probability framework; different distinct sources based on the certainty of them can be incorporated to achieve better result. Some other distinctive aspects of this space are counted in the following:

- 1) Possibility to cooperate prior information [10]
- Obtaining full probability distribution on unknown parameters presenting whole knowledge of them [11]
- 3) Using credible interval instead of confidence interval
- 4) Absence of over parameterized phenomena [13]
- Evaluate method in the presence of limited number of observed data

Besides the pros of the proposed method, its drawbacks are enumerated as follows:

- 1) High computational effort for obtaining whole probability distribution
- 2) Some assumptions on knowing noise distribution before estimation

It is true that Bayesian methods are employed for identification problems in some articles and it is somehow a mature topic (e.g. [11], [14], [15]), but it is not widely developed for environment identification of teleoperation systems. Hence, it has been a real challenge in recent literature. Firstly, in [16], Bayesian hypothesis tree is utilized to estimate location of remote object. Reference [17], reports adaptive methodology

to maintain stability without compromising performance with finite set of environment models. Specific switching policy between these models are employed by Markov chain approach. Internet communication with packet loss and data corruption by the solution of Particle filters are discussed in [18]. In [19], motion and force are estimated using two particles in master and slave side. Different application of particle filters in teleoperation with time delay is surveyed in [20]. A new method to precisely obtain time delay in teleoperation via internet is proposed in [21].

In this paper, we use particle filter with sampling importance resampling (SIR) algorithm in the slave side to filter motion and velocity signals required for local impedance control and predict force flow to use as feedback in bilateral framework. For the sake of comparison, this algorithm is collate with extended Kalman filter (EKF) method to estimate environment dynamic.

The reminder of this paper is organized as follows. In Sec. II, brief explanation of teleoperation and probability approach for filtering are presented. Sec. III is devoted to structure of proposed controller in master and slave side. Detailed algorithm is expressed in Sec. IV including EKF and SIR Particle filter. Simulation results are addressed in Sec. V. Finally, the paper is concluded in Sec. VI.

#### II. PRELIMINARIES

In this section, some precursory implications are wrapped up to declare the forgoing concepts. These intention comprise bilateral teleoperation and probabilistic perspective in nonlinear system identification case.

#### A. Bayesian approach

In Time-series modeling of state-space approach, the state vector includes all available information of a system. Two dedicated models are required to make inference about a dynamic system; first, a system model that qualify the evolution of the state as time goes on and, second, a measurement model including the noisy measurements. It is assumed that these models are available in a probabilistic form. This assumption provides a rigorous framework for dynamic state estimation problems.

Generally, the posterior probability density function (pdf) of the states includes the evolution model and received measurement to capture the whole available information in the Bayesian approach. This approach tends to the complete solution of the estimation problem, since the pdf embodies all available statistical information [22].

Similar to recursive approaches, in the state estimation problems, the algorithm should be updated every time as a measurement arrives. In a recursive filters, data processing are done sequentially and the current state is adjusted only by the latest received information regardless of batch datasets. These filters are implemented in two fundamental stages: prediction and update. In the prediction stage, the state pdf is predicted using the system model. The update operation modifies this pdf by the latest available measurement. These procedure can be employed systematically using Bayes theorem, which is the method to update evolution information given measured

data. To define the problem of tracking, consider the following evolution of the state sequence:

$$x_k = f_k(x_{k-1}, u_{k-1}, \nu_{k-1}), \tag{1}$$

where  $(f_k)$  is a possible nonlinear function of state vector  $(x_k)$  and exogenous input  $(u_k)$  and  $(\nu_{k-1})$  is i.i.d. process noise sequence. The main objective of the tracking problem is to recursively update  $(x_k)$  from noisy measurement equation:

$$z_k = h_k(x_k, u_k, n_k), \tag{2}$$

where  $(n_k)$  is i.i.d. measurement noise. Particularly, a filtered estimates based on the set of all available measurements  $(z_{1:k})$  is required in the state estimation problems. In a Bayesian point of view, the identification problem is to recursively calculate some degree of belief in the state  $(x_k)$ , given the measured data set  $(Z_{1:K})$  up to time (k). In order to achieve this end, the posterior pdf  $p(x_k|z_{1:k})$  need to be constructed in each time sample. The following assumptions should be hold:

- Some prior information of Initial pdf  $(p(x_0|z_0))$  is required to be known.
- The sequence of  $(x_k)$  is Markovian process of order one and only depends on the past observations through its own history.
- The sequence of  $(z_k)$  is first order Markovian process.

*Prediction:* It is assumed that the posterior pdf is available at the current time. In the prediction stage, the system model as in Eq. (1) and known statistics  $(v_{k-1})$  is calculated to obtain the prior pdf at current time using the Chapman–Kolmogorov equation [23]:

$$p(x_k|z_{1:k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|z_{1:k-1})dx_{k-1}.$$
 (3)

*Update:* As the measurement  $(z_k)$  becomes available at time step (k), this is used to update the prior knowledge in Eq. (3) via Bayes' rule:

$$p(x_k|z_{1:k}) = \frac{p(z_k|x_k)p(x_k|z_{1:k-1})}{p(z_k|z_{1:k-1})},$$
(4)

with the following normalizing constant:

$$p(z_k|z_{1:k-1}) = \int p(z_k|z_k)p(x_k|z_{1:k-1})dx_k.$$
 (5)

First expression in nominator of Eq. (4), is called the likelihood function derived from measurement model in Eq. (2); the next expression is the prior knowledge defined by evolution model in Eq. (1). The recursive relations (3) and (4) are obtained form the core of the optimal Bayesian solution. However, this recurrent propagation relation for obtaining the posterior density is not analytically tractable, in general, it is a conceptual solution, except under some restrictive assumptions holds on general estimation problem as the one in kalman Filter.

In Kalman filters, it is assumed that the posterior density is Gaussian at every time step and, hence, can be fully interpreted by a mean and covariance hyperparameters. Particularly, the process and measurement noises  $(\nu_{k-1},n_k)$  are drawn from Gaussian distributions. Moreover, in evolution and measurement relations,  $(f_k,h_k)$  are known linear function of state, inputs and noise.

# B. Particle Filter

Particle filters also known as sequential Monte Carlo methods are substantially a generalization of the traditional Kalman filtering methods. They are based on point mass which represent the probability densities. Particle filters can be applied to many state estimation problems. One of the main method typically applied in these filters is the sequential importance sampling (SIS) algorithm. It utilizes Monte Carlo (MC) to simulate over system model and observed data. Survival of the fittest, bootstrap filtering, interacting particle approximations, and the condensation algorithm, are some other conventional referees for sequential Monte Carlo (SMC) approach [21]. This technique is actually employed to implement an online recursive Bayesian filter via MC simulations. This method is used widespread in the area of research targeting nonlinear environment with non Gaussian noise such as in image processing and robotics. The key concept is to consider the required posterior density function with a set of random samples with associated weights. These samples with their associate weights are established to represent the posterior density discretely. This MC approach becomes an equivalent representation to the actual posterior pdf as the number of samples becomes very large, and the SIS filter conquer the optimal Bayesian estimation problem. To provide the details of the algorithm, let  $(x_{0:k}^i, w_k^i)_{i=1}^{N_s}$  denote a random measure that represent the posterior pdf  $p(x_{0:k}|x_{1:k})$ . Moreover, the weights are normalized such that  $\sum_i w_k^i = 1$ . The posterior density function can be approximated as following:

$$p(x_{0:k}|z_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(x_{0:k} - x_{0:k}^i).$$
 (6)

Meanwhile, a discrete approximation is proposed to the true posterior density. In This stage, it is assumed that  $(p(x) \propto \pi(x))$  is a target probability density function from which it is difficult to draw samples but it can be evaluated on separated points. Consider  $(q(\cdot))$  as an arbitrary density which is easy to draw sample.

$$w_k^i \propto \frac{p(x_{0:k}^i|z_{1:k})}{q(x_{0:k}^i|z_{1:k})}. (7)$$

In the recursive algorithm we have:

$$w_k^i \propto w_{k-1}^i \frac{p(z_k | x_k^i) \ p(x_k^i | x_{k-1}^i)}{q(x_k^i | x_{k-1}^i, z_k)}. \tag{8}$$

Another MC method that can be applied to recursive Bayesian filtering is the SIR filter which is proposed in [24]. Some slight assumptions need to be hold in order to use SIR algorithm; the state dynamics and measurement functions, mentioned in Eqs. (1) and (2), need to be known. The prior and process noise distribution need to be able to draw sample realizations. Finally, the likelihood function up to proportionality should be point-wise available for evaluation.

Admitting the required assumptions, the SIR algorithm is calculated directly from the SIS algorithm by appropriate choice of the importance density and resampling step.  $q(\cdot)$  as the importance density should be chosen to be the prior density of  $p(\cdot)$ . Moreover, at every time sample, resampling may need to derive in this algorithm as follows.

$$w_k^i \propto w_{k-1}^i p(z_k | x_k^i). \tag{9}$$

the importance sampling density for the SIR filter, namely  $(w_{k-1}^i=1/N)$ , is available independent of measurement. On the other words, the state-space is probed without any interposition of the observations. This will increase the sensitive to outliers and make the algorithm inefficient. Abatement of diversity in particles can be taken into account as another side effect of SIR method, since resampling is applied at every iteration [25]. On the other hand, this method takes the advantage of the capability to easily evaluated the importance weights. Therefore, importance density can be easily sampled in this approach.

# III. CONTROL STRUCTURE

The teleoperation setup used in this paper is shown in Fig. (1). Master manipulator dynamics may be represented as follows by alleviating nonlinearities.

$$j_m \ddot{\theta}_m + b_m \dot{\theta}_m + k_m (\theta_m - \theta_{m0}) = u_m + F_h, \tag{10}$$

where  $(j_m, b_m, k_m)$  are inertia, damping and friction coefficient of master robot. Moreover,  $(u_m)$  is the master controller and  $(F_h)$  is the operator applied force. Slave dynamic can be considered as another 1-DOF prismatic manipulator with following dynamic:

$$m_s \ddot{x}_s + b_s \dot{x}_s + k_s (x_s - x_{s0}) = u_s - F_e,$$
 (11)

where  $(m_s, b_s, k_s)$  are mass, damping and friction coefficient of slave robot. Moreover,  $(u_s)$  is the slave control signal and  $(F_e)$  is the environment force considered as Hunt-Crossley model [8].

$$F_e = K_e x_s^n + B_e x_s^n \dot{x}_s, \tag{12}$$

where  $(K_e)$  and  $(B_e)$  are the stiffness and damping parameters respectively related to the coefficient of restitution. It is assumed that, the viscous force is dependent on the penetration into the contact area. The parameter (n) is a constant that depends on the material and the geometric properties of contact area which is usually between 1 and 2. Control structure consist of master and slave impedance controllers. For the slave side, particle filter is utilized to filter position and velocity signals and predict environment force to feedback in master side. Slave control signal is obtained base on Eq. (13).

$$u_s = F_e + F_{exg} + F_{is}, (13)$$

where

$$F_{exg} = K_{tun} \left( \frac{m_s}{m_{ds}} \right) (K_x \bar{x}_m - x_{fs}), \tag{14}$$

$$F_{is} = \left(K_s - \frac{m_s}{m_{ds}} K_{ds}\right) x_{fs} + \left(B_s - \frac{m_s}{m_{ds}} B_{ds}\right) \dot{x}_{fs}, \quad (15)$$

in which  $(K_x, K_{tun})$  is the position scaling factor and tuning parameter, respectively. Master controller is another impedance controller base on Eq. (16).

$$u_m = -F_h + \frac{j_m}{j_{dm}} (F_h - K_f \bar{f}_e) + F_{im}$$
 (16)

where  $(K_f)$  is the scaling factor and,

$$F_{im} = (K_m - \frac{j_m}{j_{dm}} K_{dm}) x_m + (B_m - \frac{j_m}{j_{dm}} B_{dm}) \dot{x}_m.$$
 (17)

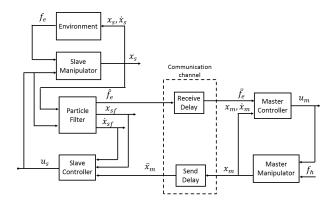


Fig. 1. Control Structure

# IV. ENVIRONMENT ESTIMATION

In this section, detailed equations are outlined for position filtering and force prediction by Kalman filter and Particle filter. Notice that, since assumption required for optimal solution dose not hold, suboptimal solution should be employed and compared in this case. Consider differential equation of slave dynamic as follows:

$$\ddot{x}_s = a_1 \dot{x}_s + a_2 x_s + bu \tag{18}$$

where  $a_1=-c_s/m_s,\ a_2=-k_s/m_s,\ b=[1/m_s,1/m_s]$  and  $u=[u_s,f_e]^T.$ 

#### A. Extended Kalman Filter

Model Equation is given by Eq. (19) as an LTV system:

$$X(k) = F(k)X(k-1) + Bu_s(k-1) + \nu_{k-1},$$
  

$$y(k) = CX(k) + n_k,$$
(19)

where  $(X = [X_1, X_2]^T)$  consist of motion and velocity data and

$$F = \begin{bmatrix} 1 & T_s \\ \frac{a_2}{a_1} (1 - e^{a_1 T_s}) + K' & e^{a_1 T_s} + B' \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & \frac{-b}{a_1} (1 - e^{a_1 T_s}) \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$
(20)

where

$$K' = -\frac{T_s}{M} n(X_1)^{n-1} (K_e + B_e X_2),$$

$$B' = -\frac{T_s}{M} B_e (X_1)^n.$$
(21)

The motion data flow is considered in a nonlinear and non-Gaussian environments in this paper. Therefore, it is assumed that the state noise  $(\nu_k)$  probability distribution can be represent by the Gamma distribution, which is typically observed in an impulse noise [26]. This density can be modeled by the following equation.

$$p(\nu_k) = \frac{\beta^{\alpha}}{(\alpha - 1)!} \nu_k^{\alpha - 1} e^{-\beta \nu_k}, \ u_k \ge 0.$$
 (22)

The mean and variance of the Gamma density function in Eq. (22) are  $(\alpha/\beta)$  and  $(\alpha/\beta^2)$ , respectively. Note that the observation noise is considered to be Gaussian. The algorithm is accomplished in three steps:

# Step 1. Initialization

Choose initial condition for X randomly.

$$X^{-} = X_0 + \epsilon, \tag{23}$$

where  $\epsilon \sim \mathcal{N}(0, \sigma_i)$ .

# Step 2. Prediction

Calculate the model equation 19.

$$X^{-} = FX(k) + Bu_s,$$
  

$$P^{-} = FP(k)F' + Q,$$
(24)

where  $\left(Q\right)$  is variance of model equations and  $\left(P\right)$  is the covariance matrix.

# Step 3. Update

Compute Kalman gain and covariance matrix.

$$K = P^{-}C'(CP^{-}C' + R)^{-1},$$

$$X(k+1) = X^{-} + K(X(k+1) - CX^{-}),$$

$$p(k+1) = (1 - KC)P^{-}.$$
(25)

# B. Particle Filter

Evolution dynamics for environment estimation based on Eq. (1), consisting of three states position, velocity and random walk for applied torque. Discreet time differential equation is given as following:

$$x^{i}(k+1) = \begin{bmatrix} 1 & T_{s} & 0\\ \frac{a_{2}}{a_{1}}(1 - e^{a_{1}T_{s}}) & e^{a_{1}T_{s}} & -\frac{T_{s}}{m_{s}}\\ K_{e}(x_{1}^{i})^{n-1} & B_{e}(x_{1}^{i})^{n} & 0 \end{bmatrix}$$

$$x^{i}(k) + \begin{bmatrix} 0\\ \frac{-b}{a_{1}}(1 - e^{a_{1}T_{s}})\\ 0 \end{bmatrix} u_{s} + \begin{bmatrix} \epsilon_{1}\\ \epsilon_{2}\\ \epsilon_{3} \end{bmatrix}$$
(26)

where  $(T_s)$  is the sampling rate,  $x^i = [x_1^i, x_2^i, x_3^i]^T$  corresponding to motion, velocity and force signals, respectively. and  $\epsilon_i \sim \mathcal{N}(0,\sigma_i)$ . These parameter  $(\sigma)$  depict degrease of belief for evolution model and must be well chosen based on preciseness on that. Meaning that if dynamic equation are highly accurate, the corresponding parameter must be chosen small enough to decrees chartering for filtered signal. It is worth mentioning that due to passive force generated by environment, the force is hold from previous sampling time. Measurement dynamic (2) is the same as in Eq. 19. Given measurement noise assumptions, the importance weight 9 is further simplified as:

$$w_{k+1}^i = e^{\frac{1}{2\sigma^2}(X - x^i)^2}. (27)$$

Moreover,  $\sigma$  is designed based on measurement noise. If it is chosen very small, only a small number of particles are active in each step. Steps to implement particle filter is expressed as bellow:

#### **Step 1. Initialization**

Choose initial condition for particles by cooperating of prior knowledge.

# Step 2. Prediction

Evaluate the evolution model based on Eq. (26) for next generation particles.

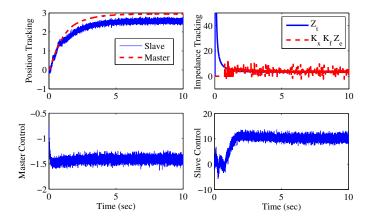


Fig. 2. Environment estimation and Control effort: extended Kalman filter

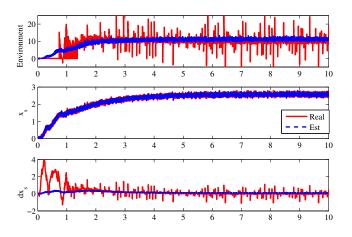


Fig. 3. Tracking of teleoperation system: extended Kalman filter

#### Step 3. Update

Calculate the normalized importance weights based on Eq. (27) based on measurement.

#### Step 4. Resampling

Multiply/suppress samples with high/low importance weights.

# V. SIMULATION

Master robot consist of 1 DOF manipulator with parameter shown in Table I. Slave robot is 1 DOF translational manipulator and environment is Hunt-Crossley model with gamma Gaussian noise. Environment position is 1.5 cm away from slave initial position. Therefore, at first the control structure must propose position tacking and then at time about 2 sec after penetration it must impose ideal force tracking. Communication channel are UDP local host having time-varying delay up to 250 msec round trip delay. Whole simulation parameters are given in Table I.

# A. Extended Kalman Filter

The tracking problem of motion, velocity and force are depicted in Fig. (2) and position and impedance tracking in teleoperation system with control signals are shown in Fig. (3). It should be noticed that, before  $x_s=1.5$  optimal

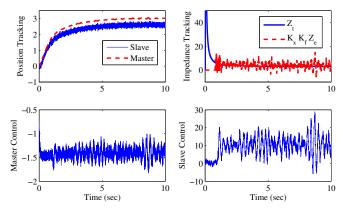


Fig. 4. Environment estimation and Control effort: Particle Filter

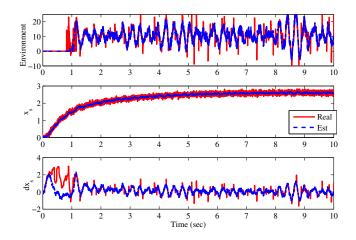


Fig. 5. Tracking of teleoperation system: Particle Filter

position tracking in free motion is required but after insertion, impedance tracking is the main objective of control structures.

### B. Particle Filter

According to Table II, there are 50 number of particles to be applicable in practice for real time implementation with 1 msec sampling rate. Estimation of environment and control signals are shown in Fig. (4). Teleoperation position and impedance tracking with control efforts are drawn in Fig. (5).

# C. Comparison

In this section, mean and variance estimation and tracking error for previously designed filters are compared. The desired variable of control parameter are listed in II. Efficiency of estimation and capability to provide suitable signals are shown in table III. As it can be seen in this table, Kalman filter mean of error is noticeable due to non-Gaussian noise result in biased estimation. Moreover, inherent nonlinearity of environment dynamic, deteriorate the resulting position and impedance tracking in the bilateral teleoperation, since predicted force and velocity are far away from real variable in Kalman filter.

TABLE I. MODEL PARAMETERS

par.	value	description	par.	value	description
$j_m$	0.05	master inertia	$m_s$	0.07	slave mass
$b_m$	0.03	master damping	$b_s$	0.05	slave damping
$k_m$	0.1	master spring	$k_s$	0.08	slave spring
$\overline{\nu}$	0.01	noise variance	$ au_d$	0.25	RTD (sec)
$K_e$	2	environment stiffness	$B_e$	3	environment viscose
$\overline{n}$	1.7	environment	$x_{e0}$	1.5	environment position

TABLE II. CONTROL PARAMETERS

par.	value	par.	value	par.	value
$j_{dm}$	0.2	$m_{ds}$	1	$K_{tun}$	300
$b_{dm}$	4	$b_{ds}$	15	$N_{par}$	50
$k_{dm}$	2	$k_{ds}$	50	$\sigma_1$	0.001
$k_f$	0.1	$k_x$	$\pi$	$\sigma_2$	0.02
$var(\nu)$	0.002	var(n)	0.005	$\sigma_3$	0.01

TABLE III. COMPARISON IN ESTIMATION AND TRACKING

Errors	EKF	PF
mean position	-0.84134	0.0058
mean velocity	0.4614	0.3049
mean force	3.3925	3.3688
mean position tracking	-0.3518	-0.3129
mean impedance tracking	-0.5616	-0.4644
RMS position	35.9604	5.8643
RMS velocity	82.8132	51.879
RMS force	616.8643	605.884
RMS position tracking	37.2563	37.8827
RMS impedance tracking	104.2703	101.4193

# VI. CONCLUSIONS

In this paper, we employ online Bayesian estimation approach for identification of nonlinear, non-Gaussian process noise model of environment to filter position and force signals required for local impedance control and predict force to feedback in bilateral teleopration for enhancement of transparency. Extended Kalman filter and particle filter as robust suboptimal solution of Bayesian problem was established and compared. Simulation results reveals that particle filter with 50 number of particles can perform fast and robust to dynamic model. Moreover, it can minimize the tracking error iteratively by well assigning parameter of the method.

# REFERENCES

- B. Siciliano and O. Khatib, Springer handbook of robotics. Springer Science & Business Media, 2008.
- [2] I. G. Polushin, A. Takhmar, and R. V. Patel, "Projection-based force-reflection algorithms with frequency separation for bilateral teleoperation," *IEEE/ASME Transactions on Mechatronics*, vol. 20, no. 1, pp. 143–154, 2015.
- [3] K. J. Kuchenbecker and G. Niemeyer, "Induced master motion in forcereflecting teleoperation," *Journal of dynamic systems, measurement, and control*, vol. 128, no. 4, pp. 800–810, 2006.
- [4] P. Agand, H. D. Taghirad, and A. Molaee, "Vision-based kinematic calibration of spherical robots," in *Robotics and Mechatronics (ICROM)*, 2015 3rd RSI International Conference on, pp. 395–400, IEEE, 2015.
- [5] L. J. Love and W. J. Book, "Force reflecting teleoperation with adaptive impedance control," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 34, no. 1, pp. 159–165, 2004.
- [6] F. H. Khabbaz, A. Goldenberg, and J. Drake, "An adaptive force reflective teleoperation control method using online environment impedance estimation," in *Robotic Intelligence In Informationally Structured Space (RiiSS)*, 2014 IEEE Symposium on, pp. 1–8, IEEE, 2014.
- [7] L. Huijun and S. Aiguo, "Virtual-environment modeling and correction for force-reflecting teleoperation with time delay," *IEEE Transactions* on *Industrial Electronics*, vol. 54, no. 2, pp. 1227–1233, 2007.

- [8] A. Haddadi and K. Hashtrudi-Zaad, "A new method for online parameter estimation of hunt-crossley environment dynamic models," in 2008 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 981–986, IEEE, 2008.
- [9] A. Haddadi and K. Hashtrudi-Zaad, "Real-time identification of huntcrossley dynamic models of contact environments," *IEEE transactions* on robotics, vol. 28, no. 3, pp. 555–566, 2012.
- [10] A. Hertzmann, "Introduction to bayesian learning," in ACM SIGGRAPH 2004 Course Notes, p. 22, ACM, 2004.
- [11] A. H. Valadkhani, A. Khormali, M. A. Shoorehdeli, H. Khaloozadeh, and A. Fatehi, "On-line full probability distribution identification of arx model parameters based on bayesian approach," *IFAC-PapersOnLine*, vol. 49, no. 7, pp. 508–513, 2016.
- [12] A. D. Martin and K. M. Quinn, "Dynamic ideal point estimation via markov chain monte carlo for the us supreme court, 1953–1999," *Political Analysis*, vol. 10, no. 2, pp. 134–153, 2002.
- [13] C. M. Bishop, "Pattern recognition," Machine Learning, vol. 128, 2006.
- [14] T. Baldacchino, S. R. Anderson, and V. Kadirkamanathan, "Computational system identification for bayesian narmax modelling," *Automatica*, vol. 49, no. 9, pp. 2641–2651, 2013.
- [15] B. Ninness and S. Henriksen, "Bayesian system identification via markov chain monte carlo techniques," *Automatica*, vol. 46, no. 1, pp. 40–51, 2010.
- [16] J. De Geeter, H. Van Brussel, J. De Schutter, and M. Decreton, "Local world modelling for teleoperation in a nuclear environment using a bayesian multiple hypothesis tree," in *Intelligent Robots and Systems*, 1997. IROS'97., Proceedings of the 1997 IEEE/RSJ International Conference on, vol. 3, pp. 1658–1663, IEEE, 1997.
- [17] S. Shahdi and S. Sirouspour, "Multiple model control for teleoperation in unknown environments," in *Proceedings of the 2005 IEEE Interna*tional Conference on Robotics and Automation, pp. 703–708, IEEE, 2005
- [18] J.-y. Lee and S. Payandeh, "Bayesian framework for bilateral teleoperation systems over unreliable network," *Robotica*, pp. 1–16.
- [19] J.-y. Lee, S. Payandeh, and L. Trajkovic, "The internet-based teleoperation: Motion and force predictions using the particle filter method," in ASME 2010 International Mechanical Engineering Congress and Exposition, pp. 765–771, American Society of Mechanical Engineers, 2010
- [20] J.-y. Lee and S. Payandeh, Haptic Teleoperation Systems. Springer, 2015.
- [21] S. Wang, B. Xu, Y. Jia, and Y.-H. Liu, "Real-time mobile robot teleoperation over ip networks based on predictive control," in 2007 IEEE International Conference on Control and Automation, pp. 2091– 2096, IEEE, 2007.
- [22] M. K. Pitt and N. Shephard, "Filtering via simulation: Auxiliary particle filters," *Journal of the American statistical association*, vol. 94, no. 446, pp. 590–599, 1999.
- [23] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking," *IEEE Transactions on signal processing*, vol. 50, no. 2, pp. 174–188, 2002.
- [24] Z. Chen, "Bayesian filtering: From kalman filters to particle filters, and beyond," *Statistics*, vol. 182, no. 1, pp. 1–69, 2003.
- [25] C. Yardim, P. Gerstoft, and W. S. Hodgkiss, "Tracking refractivity from clutter using kalman and particle filters," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 4, pp. 1058–1070, 2008.
- [26] L. Smith and V. Aitken, "The auxiliary extended and auxiliary unscented kalman particle filters," in 2007 Canadian Conference on Electrical and Computer Engineering, pp. 1626–1630, IEEE, 2007.