

Robust \mathcal{H}_∞ Control of a 2RT Parallel Robot For Eye Surgery

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Abstract—This paper aims at designing a robust controller for a 2RT parallel robot for eye telesurgery. It presents two robust controllers designs and their performance in presence of actuator saturation limits. The nonlinear model of the robot is encapsulated with a linear model and multiplicative uncertainty using linear fractional transformations (LFT). Two different robust control namely, \mathcal{H}_∞ and μ -synthesis are used and implemented. Results reveal that the controllers are capable to stabilize the closed loop system and to reduce the tracking error in the presence of the actuators saturation. Simulation results are presented to show that effectiveness of the controllers compared to that of conventional controller designs. Furthermore, it is observed that μ -synthesis controller has superior robust performance.

I. INTRODUCTION

Using minimally invasive eye surgeries is becoming very popular in advanced medical operations. This type of surgery is performed through tiny incisions, and therefore, the patients tend to recover in a shorter time and less discomfort. To make full advantage of the minimally invasive surgery on the eye, minimum force must be applied during the surgery at the span of the sclera. This requirement motivates using remote center of motion (RCM) mechanisms to perform eye surgeries. RCM point may be assured by mechanical design or by software. Most manipulators being assisted for eye surgery, provides two spherical degrees of freedom in RCM point in addition to one other degrees of freedom in line with the surgery tool axis [1], [2].

The RCM can be achieved by many different mechanisms. The robot considered in this paper is based on the spherical parallel mechanism. In this mechanism all joint axes intersect at an RCM point and the workspace of the mechanism is a hemisphere. As it is shown in Fig. 1, the manipulator has two pairs of identical spherical limbs which are serial and have common axes at both ends. All together they make a parallel robot with two degree of rotational motion. One of the significant advantages of this mechanism in eye surgery robot is its parallel structure, and inherent stiffness which makes it appropriate for precise motions such as the eye surgery application. The structure and the kinematics of the robot is presented in [3], calibration of kinematics presented in [4] and the explicit form dynamics of the manipulator is formulated by Gibbs-Appell which is so much similar to [5].

To synthesize robot controllers in the literature, \mathcal{H}_∞ and μ synthesis design techniques are reported in presence of unmodeled dynamics and parameter uncertainty [6]. In addition, one of the important practical limitations is actuator

saturation. Performance of the closed-loop system may be deteriorated by the saturation due to some limitations such as slow responses, undesirable transitions, or even instability [7]. A comparative study on different robust control methods on under-actuated manipulator is reported in [8]. Furthermore, control of a flexible joint of an industrial manipulator using \mathcal{H}_∞ design and \mathcal{H}_∞ loop shaping is presented in [9].

In this paper the aim is to design linear robust controllers for the spherical parallel manipulator in hand in presence of modeling uncertainty and actuator saturation. Since these controllers are targeted to be implemented on the real platform, stability assurance in presence of modeling uncertainty, in addition to suitable tracking performance in presence of actuator saturation are considered in the controller synthesis. In order to achieve that, the nonlinear and coupled dynamics of the robot is encapsulated to a linear model and multiplicative uncertainty in presence of parameter variations. Further more a mixed sensitivity formulation is proposed for the controller design, in order to find an appropriate trade-off between robust stability and performance in presence of actuator saturation limit. Finally two method of \mathcal{H}_∞ and μ -synthesis are implemented and compared in terms of their robustness and performance. The rest of the paper is organized as follows. The linear encapsulation of the nonlinear system is described in Section II. Section III presents optimal \mathcal{H}_∞ design for the system in the presence of actuator saturation. The controller design by μ -synthesis is discussed in Section IV. Sections V

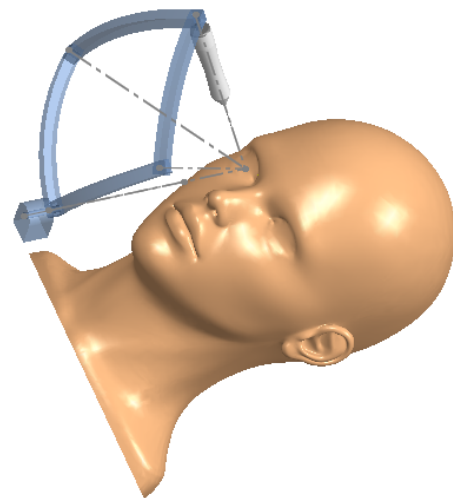


Fig. 1. 2RT parallel robot for eye telesurgery

reports simulation results and comparisons. Finally, concluding remarks are stated in Section VI.

II. LINEAR ENCAPSULATION OF NONLINEAR SYSTEM

In order to apply \mathcal{H}_∞ synthesis to the system, the nonlinear system may be encapsulated by a linear model with multiplicative uncertainty. This might not be effectively performed, in systems such as parallel robots, where nonlinear behaviors are dominant, and linear encapsulation results in large uncertainty profiles. For such system we propose to use a feedback linearized system to be used for such encapsulation. In here, we propose to have an internal feedback by use of an Inverse Dynamics Control (IDC) and a PD controller. By this means the robot performs more linearly, and such encapsulation will become more effective [10]. Fig. 2 illustrates the schematics of such implementation, and in order to verify the effectiveness of this internal feedback, a Sinusoidal input is applied to the first input while the other input is zero. The two outputs of the systems are depicted in Fig. 3. As it is seen in this figure the two outputs are totally decoupled and the feedback linearization has worked well for this manipulator.

Now, in order to identify the linear and second order model for each channel a step input is applied to each of the channels and from the system outputs, the overshoot (M_p) and the peak response (P) is determined. By this means, the natural frequency (ω_n), and the damping coefficient (ζ) is calculated and for each channel and a second order system will be nominated for each part by the following formulations:

$$G = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1)$$

in which,

$$\zeta = \frac{\ln(M_p/100)}{\sqrt{(\pi^2 + \ln^2(M_p/100))}}, \quad \omega_n = \frac{2\pi}{2P\sqrt{1 - \zeta^2}} \quad (2)$$

In order to consider parametric uncertainty in the system, and to analyze the robustness of the design, all the parameters of the robot is perturbed by 10% in the simulations. This has been implemented by the *Pert* block in the simulations as illustrated in Fig. 2. By changing perturbation coefficients, fifteen different models for the system is obtained, and one of the system outputs is shown in Fig. 4, as a representative. Among these uncertain models, one which is nearly the median of all responses is considered as the nominal model and is

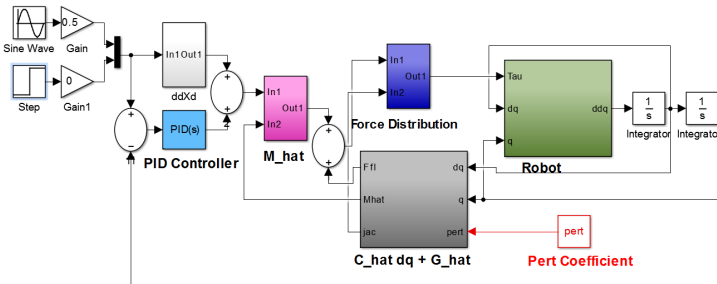


Fig. 2. Simulink model of the robot with IDC controller

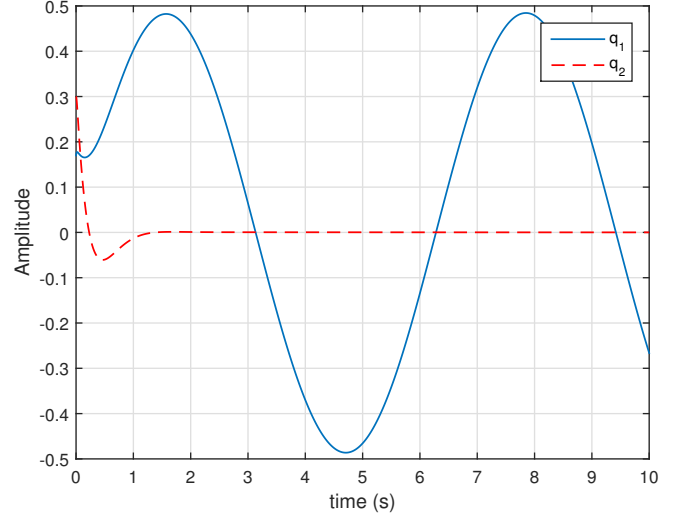


Fig. 3. System response with IDC controller

given for each channel as follows: Fig.4.

$$G_1 = \frac{25.86}{s^2 + 9.058s + 32.32}, G_2 = \frac{25.47}{s^2 + 8.432s + 25.47} \quad (3)$$

Next step is to encapsulate the uncertainty of the system. We propose to use a full block of multiplicative uncertainty instead of many different source of parametric and unmodeled dynamics for such systems. For this purpose, nominal model depicted by G_o is related to the perturbed system denoted by G_p through a permissible and normalized uncertainty block Δ , in which ($\|\Delta\|_\infty < 1$), with the uncertainty weighting function W [11], as follows:

$$G_p = (1 + \Delta W)G_o. \quad (4)$$

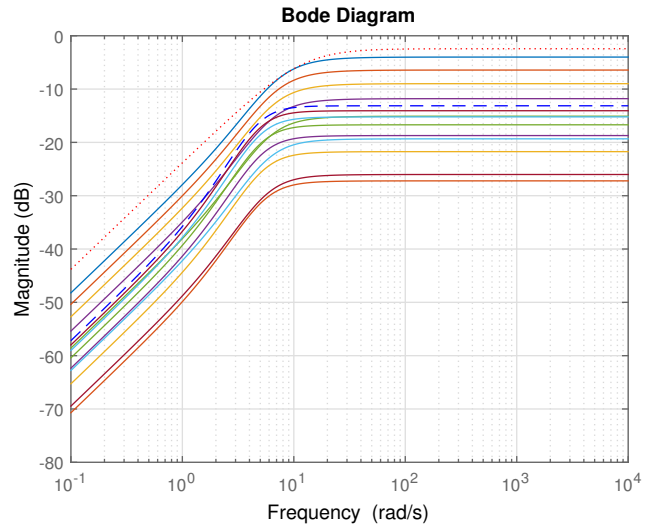


Fig. 4. Multiplicative uncertainty profiles for the first channel

One may obtain the uncertainty profile by:

$$\left| \frac{G_p(j\omega)}{G_o(j\omega)} - 1 \right| < |W(j\omega)|, \quad \forall \omega. \quad (5)$$

performing this calculations for fifteen different perturbed model in each channel, the uncertainty profile is calculated, and is illustrated in Fig. 4, as a representative. The least upper bound of these perturbations may be nominated as the weighting functions for each channel, by the following transfer functions:

$$W_1 = \frac{0.7556(s + 0.0017)}{(s + 11.71)}, W_2 = \frac{0.4228(s + 0.0024)}{(s + 9.225)} \quad (6)$$

III. ROBUST \mathcal{H}_∞ CONTROLLER

The aim of this part of the paper is to design a robust \mathcal{H}_∞ controller for the two degree of freedom robot. Objectives of the optimal controller design is to achieve to robust stability, while having a suitable tracking performance in presence of modeling uncertainty and actuator saturation. These objectives may be well reduced to the following mixed sensitivity problem as represented by the block diagram depicted in Fig. 5:

$$\|T_{zyd}\|_\infty = \left\| \begin{bmatrix} W_s S \\ W_u U \\ W T \end{bmatrix} \right\|_\infty < 1. \quad (7)$$

In which, for a multiplicative uncertainty representation, $\|WT\|_\infty < 1$ is the result of small gain theorem to preserve robust stability. Furthermore, through considering suitable performance weighting function W_s and enforcing $\|W_s S\|_\infty < 1$ the tracking performance is obtained. By including $\|W_u U\|_\infty < 1$ in the optimization cost function the effect of actuator saturation will be represented in the optimal design problem.

Selection of $W_u(s)$ and $W_s(s)$ shall be done simultaneously in order to achieve the required tracking performance in presence of actuator saturation limits. First select $W_u(s)$ as constant level $W_u(s) = \alpha$, by which the actuator effort is reduced to $\|U(s)\| < 1/\alpha$ for all frequencies. The level of α is selected through closed-loop simulations to avoid reaching to the saturation limit. Furthermore, by increasing the actuator weighting function $W_u(s)$ at high frequencies, the high frequency content of the control action (fast transients and jumps) will be decreased [7].

To select the performance weighting function, the desired input-output behavior for both channels are assumed as a standard quadratic transfer function based on a suitable step

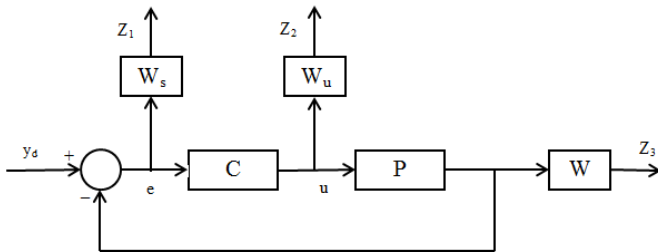


Fig. 5. Block diagram representation of mixed sensitivity problem for the system

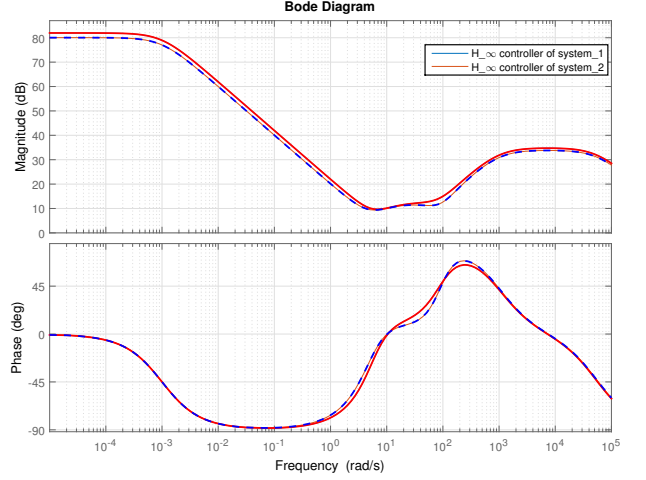


Fig. 6. Bode plot of the designed \mathcal{H}_∞ controllers

response requirement. For instance, one may require a desired settling time less than one seconds and a desired overshoot of less than %20. Thus, from $\frac{4.6}{\zeta \omega_n} < 1$, $\exp\left(\frac{-\zeta \pi}{\sqrt{1-\zeta^2}}\right) < 0.2$, one may obtain $\zeta = 0.5$, $\omega_n = 10 \text{ rad/s}$, and hence the desired closed loop system transfer function to be:

$$T_{id} = \frac{100}{s^2 + 10s + 100}. \quad (8)$$

Hence,

$$S_{id} = 1 - T_{id} = \frac{s(s + 10)}{s^2 + 10s + 100} \quad (9)$$

Now, with the aim of ensuring $\|W_s S\|_\infty < 1$, W_s can be achieved:

$$W_s < \frac{1}{S_{id}} = \frac{s^2 + 10s + 100}{s(s + 10)}. \quad (10)$$

In order to make W_s strictly proper and belonging to the $R\mathcal{H}_\infty$ space, one may add a far pole to it and slightly perturb the pole at the origin. Furthermore, assume a coefficient a to the final form of the weighting function, in order to adjust the performance in the design procedure.

$$W_s = a \frac{s^2 + 10s + 100}{(s + 0.001)(s + 10)(0.001s + 1)} \quad (11)$$

First assume α , and a equal to one and solve the mixed sensitivity problem for each channel by robust toolbox of matlab¹ to generate the decentralized controller for each channel. Then the controller are simulated in the closed loop for the robot, and the tracking performance and the control effort is examined. The design procedure will be adjusted iteratively to reach to the desired performance in presence of actuator limits, The final values obtained for the control effort design parameter α is 0.018 and 0.02, and furthermore for the performance parameter a is set to 1 for both channels. Finally, the bode plot of the robust \mathcal{H}_∞ controllers for both channels are given in Fig. 6.

¹The hinfsyn command is used.

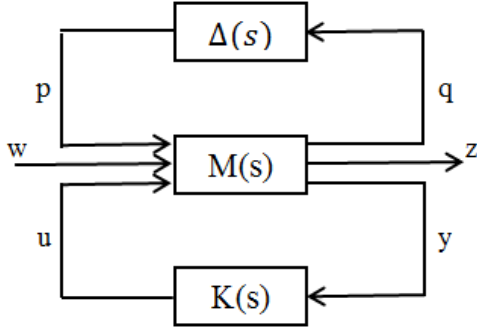


Fig. 7. Standard representation of $\Delta - M - K$

IV. THE μ -SYNTHESIS CONTROLLER

Although the robust \mathcal{H}_∞ controller may perform well in practice, iterative approach to reach to the suitable controller is manual and needs design experience. In order to automate the design iterations, one may use μ -synthesis in the design procedure, which is represented in this section. In this controller design framework the optimization problem is defined based on upper and lower linear fractional transformations. The uncertainty block shall be extracted as an upper LFT, while the controller is represented as a lower LFT to the generalized plant transfer matrix represented by M as depicted in Fig. 7. By this means not only robust stability is considered in the optimization problem, but the required performance of the system is also considered in the presence of the upper block of uncertainty. Such problem may be written in terms of LFT's as follows:

$$\{F_u(M, \Delta) : \Delta \in \Delta, \max_{\omega} [\Delta(j\omega)] \leq 1\} \quad (12)$$

In which, Δ is the set of all stable and permissible uncertainty, whose its infinity norm is less than one. In this problem, the aim of controller design is to find a stabilizing controller K such that for all Δ member of Δ , the infinity norm of the system closed loop transfer function is minimized. This goal may be interpreted using minimization of the following lower LFT.

$$\min_{\gamma} \|F_l[F_u(M, \Delta), K]\|_{\infty} < 1 \quad (13)$$

That is equivalent to $\min_{\gamma} \|T_{zw}\|_{\infty} < 1$. This objective may be reached by calculation of the structural singular value of the system as follows:

$$\max_{\omega} \mu_{\Delta}(F_l(M, K)(j\omega)) < 1 \quad (14)$$

This optimization problem is iteratively solved in the robust toolbox of matlab². The only drawback of this procedure is high order controllers that are obtained due to the nature of solving this problem. In order to apply this solution to our problem in hand, the first step is to generate the uncertain system, including the optimization objectives as the closed loop transfer function. This setup is well done in previous section by encapsulation of the system by linear model and multiplicative uncertainty, and defining the performance trade-off in presence of actuator saturation. The same weighting functions for performance as well as actuator efforts are

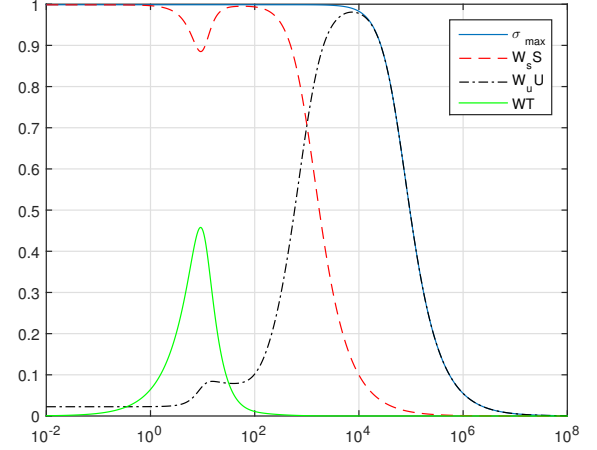


Fig. 8. The closed loop system singular values for $W_u = 0.018$, $a = 1$ for first channel

considered in this case as that given in \mathcal{H}_∞ design procedure, while the initial adjusting coefficient for α is set to 0.018 and 0.02, respectively for the two channels as before, and that for a is set to 0.75 for the performance coefficient. The obtained controllers and their closed loop performance will be elaborated in the following section.

V. SIMULATION STUDIES

The final \mathcal{H}_∞ controller design reach to the optimal performance index of $\gamma_{opt} = 0.9987$ and $\gamma_{opt} = 0.9953$, respectively for two channels. First frequency analysis of the closed loop system is reported. For this reason, the singular value plot of the system in first channel is depicted in Fig. 8, while the plot for the other channel are very similar, and therefore, not reported in here. The solid blue line in this figure depicts the final closed loop maximum singular value, which is quite flat for a large frequency range and its maximum value is less than one. The solid green curve shows the bode diagram of the WT transfer function, whose maximum value is about 0.45. Consequently, the closed loop system is robustly stabilized with a margin of greater than two. Furthermore, the control effort transfer function is plotted by dash-dotted black curve. This curve is relatively small at low frequencies, while increasing at higher frequencies, as expected. However, its infinity norm is smaller than one. Finally, the performance transfer function $W_s S$ is plotted by dashed red line. As it is seen this curve is close to one at low frequency and reduces first at frequencies, where robust stability curve peaks. This ensures performing well while preserving robust stability.

In order to verify the time response of the closed loop system, the nonlinear system is used with the final \mathcal{H}_∞ controller in closed loop. Furthermore, the time response of the closed loop system is examined for different fifteen uncertain models. To study the performance of the system unit step response is examined, while a saturation block is added to the control effort signal. The step response and its required control effort is shown in Fig. 9. As it is shown in the upper figure, the closed loop tracking performance is very suitable. The closed loop systems overshoot is less than 20% and the

²Function `musyn` will perform this task.

settling time is less than one seconds. Furthermore, from the lower figure can be concluded that, control effort amplitude high at first few moments and then will be reduced. However, high frequency oscillations are observed in the control effort:

Next Consider the μ -synthesis controller design for the system. The procedure is done for three iterations, while the initial structured singular value of 1.197 is reduced to 0.892 for the first channel. This is slightly different for the second channel, in which through four iterations the structured singular value of 1.072 is reduced to 0.833. The order of the final controllers are 27, and 24, for the first and second channels respectively. In order to reduce the order of the controller, their Hankel norm is plotted as in Fig. 10, and it is observed that the truncation may be done suitable to the order of four. The final reduced order controller are given as follows:

$$C_{\mu_1}(s) = \frac{33730(s + 0.03857)(s^2 + 12.95s + 43.93)}{(s + 1153)(s + 116.3)(s + 0.0406)(s + 0.001)} \quad (15)$$

$$C_{\mu_2}(s) = \frac{81717(s + 108.9)(s + 1.139)(s + 0.0015)}{(s + 2381)(s + 467.1)(s + 0.00153)(s + 0.001)} \quad (16)$$

Now, the time response for this controller is analyzed as that for the \mathcal{H}_∞ controllers. The step response of the closed loop system with μ -synthesis reduced order controllers, and its required control effort for fifteen uncertain systems are shown in Fig. 11. As it is shown in the upper figure, the closed loop tracking performance is very suitable. The closed loop systems overshoot is less than 20% and the settling time is less than one seconds, as required. Moreover, the control effort amplitude is high at first few moments and then will be reduced, while no oscillations are observed compared to that of \mathcal{H}_∞ controllers. Comparing the time responses of the two controllers, it is observed that the μ -synthesis controller performs better in terms of control effort for uncertain systems.

In order to compare the two sets of controllers by a quantitative measure, the robust performance of the resulting

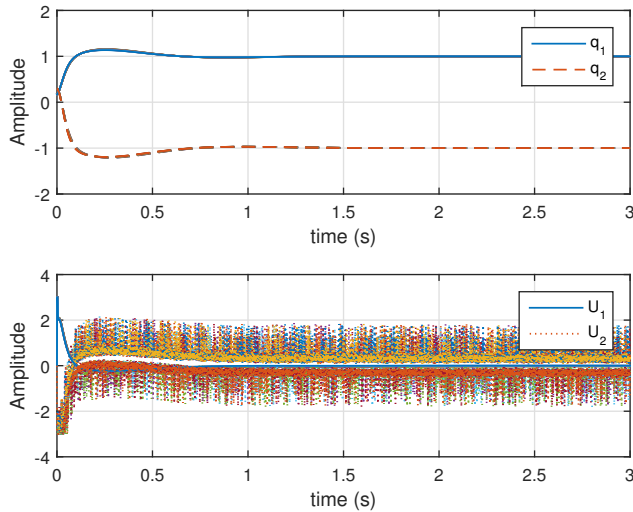


Fig. 9. The unit step response of the uncertain nonlinear closed loop systems with \mathcal{H}_∞ controller and the required control effort for fifteen uncertain models

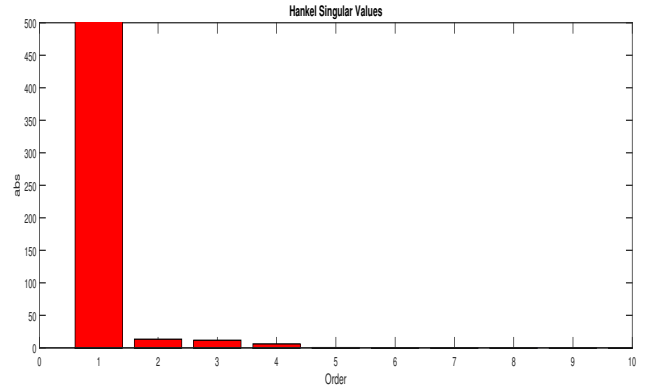


Fig. 10. Hankel norm plot of the μ -synthesis controller for the first channel.

controllers in both channels are studied in detail. For this purpose, the structured singular values for both controllers, and both channels are depicted in Fig. 12. As it is shown in this figure, the reduced order μ -synthesis controllers meet the robust performance criterion for all frequencies, and their structured singular values are lower than one. The \mathcal{H}_∞ controllers do not meet the robust performance criterion for all frequencies, and their structured singular values are more than one in particular frequencies.

Next, the frequency response of the resulting controllers are shown in Fig. 13. As it can be seen in this figure, the controllers are similar to each other at low and mid frequencies, and their performance are close to each other. However, at high frequencies the \mathcal{H}_∞ controller magnitude is more than that of μ -synthesis controller, and this has led to more oscillatory control effort. Therefore, the μ -synthesis controller has showed better performance.

Finally, the performance of the uncertain closed loop system with PID controller is compared to that of the reduced

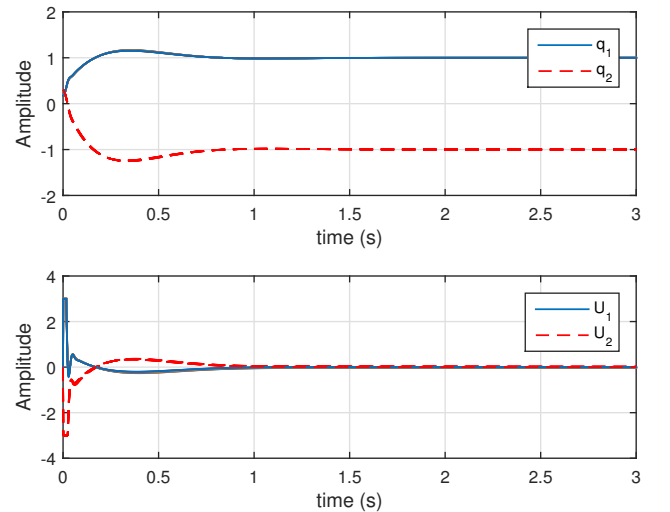


Fig. 11. The unit step response of the uncertain nonlinear closed loop systems with reduced order μ -synthesis controller and the required control effort for fifteen uncertain models

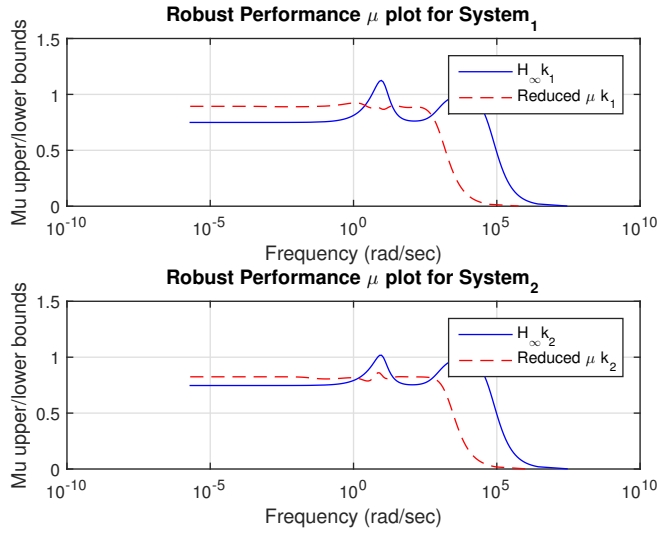


Fig. 12. Robust performance analysis of two designed controllers for the both SISO systems

order μ -synthesis ones for two channels. As it is seen in Fig. 14 the performance of the reduced order μ -synthesis controller is much better than that of the PID controller, in terms of absolute tracking error, overshoot, and settling time.

VI. CONCLUSIONS

In this paper two robust control strategies, namely \mathcal{H}_∞ and μ -synthesis is proposed for the 2RT parallel robot. In order to design such controllers, the nonlinear model of the robot is encapsulated with a linear model and multiplicative uncertainty. A mixed sensitivity objective is optimized for the \mathcal{H}_∞ controllers, in which robust stability is preserved while a tradeoff between performance and limited actuator efforts are given. The tracking performance of both controllers are very suitable, and close to the desired one. However, the control

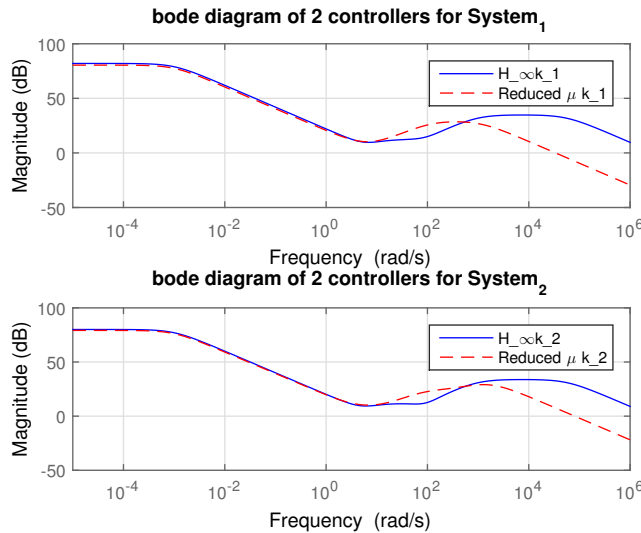


Fig. 13. Frequency response of two resulting controllers for both channels

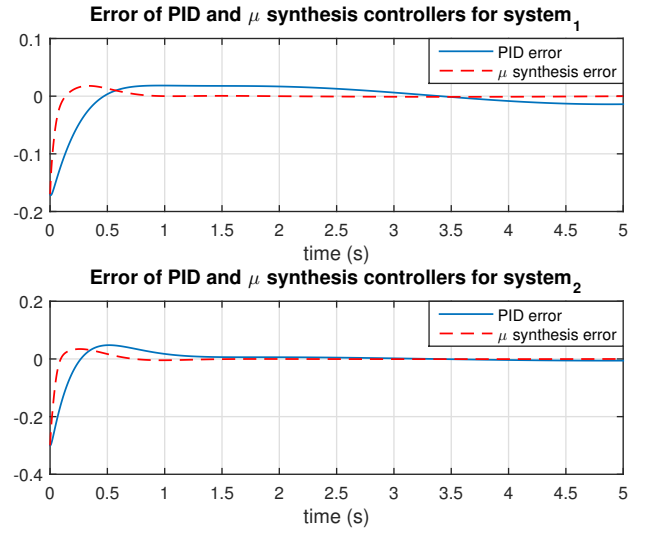


Fig. 14. Comparison of the tracking errors of PID and μ -synthesis controllers

effort of μ -synthesis controller is more preferable, and does not require any high frequency content. The proposed controllers are promising to be used in real implementation, since the resulting controllers are linear and low order, hence, easily implementable.

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