Point-to-Point Motion Control of An Underactuated Planar Cable Driven Robot

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***Abstract*—Despite being vastly investigated for underactuated serial robots, there is less attention paid to controller design of underactuated parallel robots in the literature. This paper studies regulation of an underactuated planar parallel cable-driven robot with three degrees of freedom. First, kinematic and dynamic equations of the robot are derived, then equilibrium points of the robot are investigated. By analyzing dynamics of the under–constrained robot, it is shown that zero dynamic is oscillatory, thus a control law is proposed by a composition of damping injection and sliding mode control. One of the greatest advantages of the proposed controller is that after convergence of the robot to the desired point, which is proven by Lyapunov theorem, it is chattering free. Finally, simulation results shows the effectiveness of the controller in practice.**

***Index Terms*—cable driven parallel manipulator, underactuated robot, regulator, damping injection.**

I. Introduction

Cable Driven Parallel Manipulator (CDPM) is a mechanism where the End-Effector(EE) is connected to the fixed frame by the means of several cables which are controlled by motors. Large work space, great speed and high acceleration are just a few adventures of these robots. Moreover, because of cable's low weight, the actuators require forces of lower magnitude in order to control the EE. However, cables can only pull, and therefore, in many applications they contain more actuators in numbers than the required Degrees Of Freedom (DOF) to enable ensuring positive tension in all cables [1]. This needs accurate kinematic parameters which makes CDPMs less enticing [2].

In order to conquer this drawback and expand the range of applications of cable-driven robots, underactuated structures have recently been the focus of researchers. Underactuated systems have fewer inputs than degrees of freedom and as a consequence, they can track special trajectory which complies with their dynamic equations. Fortunately point to point motion in which the trajectory between two points is undefined can be implemented [3], [4]. Therefore, cable-driven robots such as one with a pick and place application have both advantages and are agile, light and also have simple mechanism and fewer actuators.

Regulation control of underactuated robots is a challenging issue for researchers. There is not a global method to control every robot of this kind. Partial Feedback Linearization (PFL) which was proposed by Spong in [5] is one of the basic tools to control underactuated systems. Olfati Saber has classified some benchmark underactuated systems into eight classes with respect to specific features of shape variables and external variables [6]. This method is based on order reduction and leads to some special structures. Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) is an alternative approach which was proposed by Ortega in [7]. In this method the dynamic equation of the robot is written in port-Hamiltonian form, then a new kinetic and potential energy is assigned to closed loop equation and by adding suitable interconnections and damping terms, asymptotic stability is ensured. The disadvantage of this method is that it leads to partial differential equations which are intractable in solution. Controlled Lagrangian method based on Lagrange equation which is similar to port-Hamiltonian method was presented in [8] and [9]. Because of the difficulty in control of underactuated robots, most of the papers focus on a specific robot. For example, cart-pole system [10], acrobat [11], VTOL aircraft [12] and underactuated serial robots [13]–[16].

In contrast to serial robots, fewer researchers consider design or control of underactuated parallel robots. Lefrançois and Gosselin have studied design and motion control of a planar serial cable–driven robot [17]. The control algorithm is based on the use of dynamic equations to plan the trajectory using the natural frequency of the system. In [18] a 2-DOF cable-driven, pendulum-like robot was considered. The control algorithm is based on an off-line trajectory planning, which requires specific initial conditions. Motion planing for a suspended mass from a car was proposed in [19]. In [20] design and optimal control of an underactuated cable-driven micro-macro robot is presented. Motion control of 3-DOF underactuated planar cable-driven robot was proposed in [21] and [22]. Sinusoidal and polynomial functions are generated for desired value of cable's length respectively. None of the mentioned papers, has presented no proof of convergence.

In this paper regulation control of an underactuated suspended cable-driven robot is proposed. This planar robot is under–constrained, this means the EE preserves some freedom once actuators are locked. The mechanism is similar to the one proposed in [21] and [22]. First, a simple model is exploited by ignoring cable dynamics. Then positions in which the robot can stay with constant inputs are derived. The main

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contribution of this paper is to develop a Lyapunov based controller to ensure convergence to desired values. Based on the behavior of zero dynamics, the control law is designed by composition of PFL, sliding mode control and damping injection in IDA-PBC method; which leads to asymptotic stability. The major advantage of this work is that the proof of convergence is presented, as well as the control law being chattering free.

The structure of this paper is as follows. Section II presents kinematic, Jacobian and dynamic equations of the robot. Equilibrium points of the robot, where the position remains unchanged by constant inputs, are exploited in section III. Section IV presents control law and proof of asymptotic stability. In section V utilizing some simulations, it is shown that the proposed control algorithm is able to regulate the robot to a desired value with a chattering free control law. Finally conclusion and future works are presented in Section VI.

II. Kinematic, Jacobian And Dynamic Analysis

Fig. 1 illustrates the underactuated planar cable driven robot and the notations that are used in the kinematic analysis. As mentioned before, this robot has two cables which are controlled by motors. The EE has two translational and one rotational degree of freedom which are determined by $[x, y]^T$ and θ, respectively, thus the robot has 3-DOF with two actuators.

Loop closure equation of the robot is as follows

$$\overrightarrow{P} = l_1\overrightarrow{\hat{s}_1} - \overrightarrow{E_1} = \overrightarrow{B} + l_2\overrightarrow{\hat{s}_2} - \overrightarrow{E_2} \tag{1}$$

where $\overrightarrow{P} = [x, y]^T$ is the position vector of EE, $\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2$ are direction of cables, $\overrightarrow{E_1}, \overrightarrow{E_2}$ are the position of cable's attachment points to EE, l_1, l_2 are length of cables and $\overrightarrow{B} = [b, 0]^T$. Eq. (1) can be rewritten in the following form

$$\begin{aligned} x &= l_1\cos(\alpha_1) + a\cos(\theta) = b + l_2\cos(\alpha_2) - a\cos(\theta) \\ y &= l_1\sin(\alpha_1) + a\sin(\theta) = l_2\sin(\alpha_2) - a\sin(\theta) \end{aligned} \tag{2}$$

Inverse kinematic and direction of cables are obtained using the following equations

$$\begin{aligned} & l_1 = \sqrt{(x - a\cos(\theta))^2 + (y - a\sin(\theta))^2} \\ & l_2 = \sqrt{(x - b + a\cos(\theta))^2 + (y + a\sin(\theta))^2} \\ & \cos(\alpha_1) = \frac{x - a\cos(\theta)}{l_1} \qquad \sin(\alpha_1) = \frac{y - a\sin(\theta)}{l_1} \\ & \cos(\alpha_2) = \frac{x - b + a\cos(\theta)}{l_2} \qquad \sin(\alpha_2) = \frac{y + a\sin(\theta)}{l_2} \\ & \hat{\mathbf{s}}_\mathbf{1} = \begin{bmatrix} \cos(\alpha_1) \\ \sin(\alpha_1) \end{bmatrix} \qquad \hat{\mathbf{s}}_\mathbf{2} = \begin{bmatrix} \cos(\alpha_2) \\ \sin(\alpha_2) \end{bmatrix} \end{aligned} \tag{3}$$

Forward kinematic has infinite solutions, since the number of equations is less than that of unknowns.

In order to derive the Jacobian matrix, time derivative of Eq. (1) is obtained:

$$\mathbf{v} = l_1\dot{\hat{\mathbf{s}}}_1 + \dot{l}_1\hat{\mathbf{s}}_1 - \dot{\mathbf{E}}_1 = l_2\dot{\hat{\mathbf{s}}}_2 + \dot{l}_2\hat{\mathbf{s}}_2 - \dot{\mathbf{E}}_2 \tag{4}$$

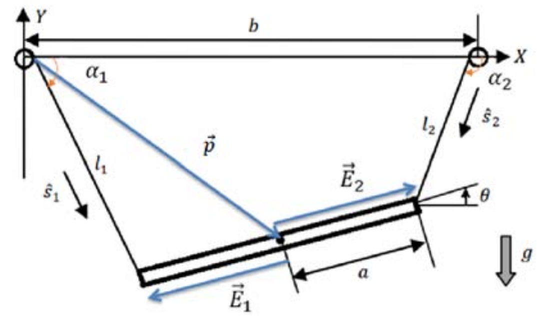


Fig. 1. Underactuated Planar cable-driven robot

which is equal to

$$\begin{aligned} \dot{l}_1\hat{\mathbf{s}}_1 &= \mathbf{v} - l_1\dot{\alpha}_1\mathbf{k} + \dot{\theta}(\mathbf{k} \times \mathbf{E}_1) \\ \dot{l}_2\hat{\mathbf{s}}_\mathbf{2} &= \mathbf{v} - l_1\dot{\alpha}_2\mathbf{k} + \dot{\theta}(\mathbf{k} \times \mathbf{E}_2) \end{aligned} \tag{5}$$

where $\mathbf{k} = [0, 0, 1]^T$. Dot multiplying the above equation by $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$, the corresponding Jacobian matrix is found.

$$\begin{bmatrix} \dot{l}_1 \\ \dot{l}_2 \end{bmatrix} = \mathbf{J}(\mathbf{X}) \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$\mathbf{J}_{2\times 3} = \begin{bmatrix} \frac{x - a\cos(\theta)}{l_1} & \frac{y - a\sin(\theta)}{l_1} & \frac{-\cos(\theta)(y - a\sin(\theta))}{l_1} + \frac{\sin(\theta)(x - a\cos(\theta))}{l_1} \\ \frac{x - b + a\cos(\theta)}{l_2} & \frac{y + a\sin(\theta)}{l_2} & \frac{\cos(\theta)(y + a\sin(\theta))}{l_2} - \frac{\sin(\theta)(x - b + a\cos(\theta))}{l_2} \end{bmatrix} \tag{6}$$

Dynamics formulation of the robot is given by [21]:

$$\mathbf{M(X)\ddot{X} + C(X, \dot{X})\dot{X} + G(X) = F = J^T(X)}\boldsymbol{\tau} \tag{7}$$

$$\mathbf{X} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix} \quad \mathbf{C} = 0 \quad \mathbf{G} = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix} \tag{8}$$

In the above equation it is assumed that cables are massless and infinitely stiff.

Equilibrium Points(EP) of a robot are obtained by solving equation $\mathbf{G(X) = 0}$. In our robot, $\mathbf{G}$ is constant and therefore it doesn't have any EP. In the next section we extend the notion of EP to a general case and then a regulation control law based on partial feedback linearization and damping injection will be proposed.

III. Equilibrium Points of The Robot

In this section a general definition for EP of a nonlinear system is proposed.

Definition 1. *Consider nonlinear input affine system in the form $\dot{x} = f(x) + g(x)u$, then x^* is an equilibrium point if*

$$f(x^*) + g(x^*)u^* = 0$$

in which u^* is a constant and also a function of x^*.

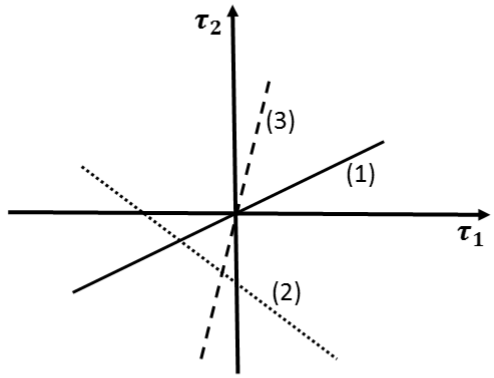


Fig. 2. Geometric description of Eq.(9)

We know that in EP, velocity and acceleration of the robot are zero. Thus according to definition 1, in EP the following equation must be held.

$$\mathbf{G} = \mathbf{J}(\mathbf{X}^*)^T \boldsymbol{\tau}^* \tag{9}$$

It is clear that equation (9) represents three equations with two variables τ_1^*, τ_2^*. The geometric description of this equation is shown in Fig.2. Two of the equations pass through the origin whereas the other one has a bias. For Eq.(9) to have a solution, the slope of the first and third equations must be equal. Therefore, the following equation is obtained.

$$\begin{aligned} &\cos(\theta^*)\big(2x^*y^* - by^* + ab\sin(\theta^*) - a^2\sin(2\theta^*)\big) = \\ &2\sin(\theta^*)\big({x^*}^2 - bx^* + ab\cos(\theta^*) - a^2\cos^2(\theta^*)\big) \end{aligned} \tag{10}$$

On the manifold shown in Fig.3, (Eq.9) has two equations with two variables, thus it is solvable if determinant of the first and second row (or equivalently second and third row) of J^T is not equal to zero. Therefore $\mathbf{X}^*$ is an EP of the robot if

- $\mathbf{X}^*$ satisfies Eq. 10
- $\det[\mathbf{J}(1), \mathbf{J}(2)] \neq 0$ ($\det[\mathbf{J}(2), \mathbf{J}(3)] \neq 0$) where $\mathbf{J}(i)$ is ith column of $\mathbf{J}(\mathbf{X})$

IV. Controller Design

In this section a regulator is proposed based on composition of PFL [23] and Sliding Controller (SC) [1] As explained before, underactuated robots can track only specific trajectories which comply with their dynamic equations, however, since these kind of robots are capable of moving from any point to a desired point, regulator design is chosen as the subject of this paper.

Since there are fewer actuators than the DOF, all of the configuration variables can not be controlled simultaneously, resulting in the necessity for studying the behavior of internal dynamic. Based on stability (not asymptotic stability) of zero dynamics, a damping injection term is added in order to achieve asymptotic stability at equilibrium point $\mathbf{X}^* = [x^*, y^*, \theta^*]^T$.

At the first stage only $\mathbf{X}' = [y, \theta]^T$ is controlled. The control law is as follows

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix} \frac{y-a\sin(\theta)}{l_1} & \frac{-\cos(\theta)(y-a\sin(\theta))}{l_1} + \frac{\sin(\theta)(x-a\cos(\theta))}{l_1} \\ \frac{y+a\sin(\theta)}{l_2} & \frac{\cos(\theta)(y+a\sin(\theta))}{l_2} - \frac{\sin(\theta)(x-b+a\cos(\theta))}{l_2} \end{bmatrix}^{-T} \begin{bmatrix}\tau_1'\\ \tau_2'\end{bmatrix} \tag{11}$$

in which τ_1', τ_2' are new inputs. By replacing (11) in (7) dynamic equation of $\mathbf{X}'$ is given by:

$$\begin{aligned} m\ddot{y} + mg &= \tau_1' \\ I\ddot{\theta} &= \tau_2' \end{aligned} \tag{12}$$

Define

$$\boldsymbol{\nu} = \dot{\mathbf{X}}_d' + \boldsymbol{\Gamma}\tilde{\mathbf{X}}', \quad \tilde{\mathbf{X}}' = \mathbf{X}' - \mathbf{X}'^*. \tag{13}$$

Then, the sliding surface is as follows

$$\mathbf{s} = \dot{\mathbf{X}}' - \boldsymbol{\nu} = \dot{\tilde{\mathbf{X}}}' - \boldsymbol{\Gamma}\tilde{\mathbf{X}}' \tag{14}$$

where $\boldsymbol{\Gamma} \in \mathbb{R}^{2\times 2}$ is a PD matrix. The new inputs are designed as follows

$$\begin{aligned} \begin{bmatrix}\tau_1'\\ \tau_2'\end{bmatrix} &= \mathbf{M}'(\mathbf{X}')\dot{\boldsymbol{\nu}} + \mathbf{C}(\mathbf{X}', \dot{\mathbf{X}}')\boldsymbol{\nu} + \mathbf{G}(\mathbf{X}') - \mathbf{K}\mathbf{s} - K'\mathrm{sgn}(\mathbf{s}) \\ &= \begin{bmatrix} m & 0\\ 0 & I\end{bmatrix}\dot{\boldsymbol{\nu}} + \begin{bmatrix} mg\\ 0\end{bmatrix} - \mathbf{K}\mathbf{s} - k'\mathrm{sgn}(\mathbf{s}) \end{aligned} \tag{15}$$

in which $\mathbf{K}$ is PD and k' is a positive integer. Assume that the exact values of kinematic and dynamic parameters in control laws (11) and (15) are available. Replacing (15) in (12) closed loop dynamic is

$$\begin{aligned} \begin{bmatrix} m & 0\\ 0 & I\end{bmatrix}(\ddot{\mathbf{X}}' - \dot{\boldsymbol{\nu}}) + \mathbf{K}\mathbf{s} + k'\mathrm{sgn}(\mathbf{s}) &= \mathbf{M}'\dot{\mathbf{s}} + \mathbf{K}\mathbf{s} \\ &+ k'\mathrm{sgn}(\mathbf{s}) = 0 \end{aligned} \tag{16}$$

Consider the Lyapunov function $V = \frac{1}{2}\mathbf{s}^{\mathbf{T}}\mathbf{M}'\mathbf{s}$, time derivative of V is

$$\dot{V} = \mathbf{s}^{\mathbf{T}}\mathbf{M}'\dot{\mathbf{S}} = -\mathbf{s}^{\mathbf{T}}\mathbf{K}\mathbf{s} - k'|\mathbf{s}| \tag{17}$$

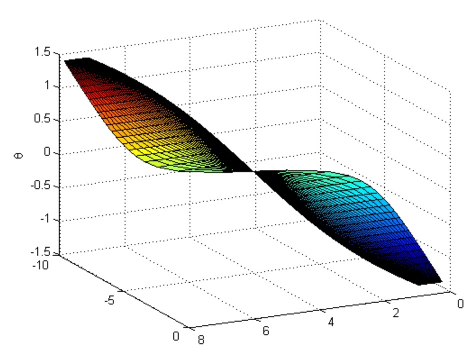
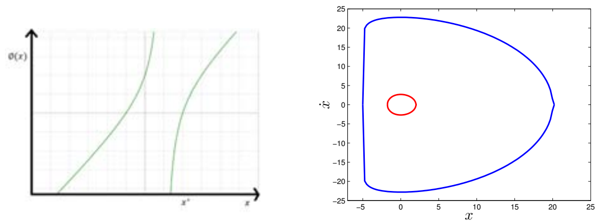


Fig. 3. Manifold of possible equilibrium points of the robot referring to definition 1 with $a = 0.25, b = 7.9$.



(a) The overall form of the function $\phi(x)$

(b) Two level sets of Lyapunov function in Eq.(21) with $\phi(x) = \frac{x(x+10)}{x+5}$. Blue and red curves are related to initial conditions $x_0 = 20$ and $x_0 = 2$ respectively.

Fig. 4. General form of $\phi(x)$ and level sets of $V(x)$.

It is clear that s converges to zero, resulting in the convergence of $\tilde{\mathbf{X}}'$ to zero.

Now investigate internal dynamics of the closed loop system. Internal dynamic of configuration variable x is determined by substituting (11) in (7):

$$\ddot{x} = \begin{bmatrix} \frac{x-a\cos(\theta)}{l_1} \\ \frac{x-b+a\cos(\theta)}{l_2} \end{bmatrix}^T \cdot \begin{bmatrix} \frac{y-a\sin(\theta)}{l_1} & \frac{-\cos(\theta)(y-a\sin(\theta))+sin(\theta)(x-a\cos(\theta))}{l_1} \\ \frac{y+a\sin(\theta)}{l_2} & \frac{\cos(\theta)(y+a\sin(\theta))-\sin(\theta)(x-b+a\cos(\theta))}{l_2} \end{bmatrix}^{-T} \begin{bmatrix} \tau_1' \\ \tau_2' \end{bmatrix} \tag{18}$$

Zero dynamics is obtained from above equation by substituting $y = y^*, \theta = \theta^*$ and $\tau_1'^* = mg, \tau_2'^* = 0$. After some manipulation one may find zero dynamics as:

$$\ddot{x} = \frac{-g\zeta(x)}{\eta(x)} \tag{19}$$

where

$$\begin{aligned} &\zeta(x) = 2\sin(\theta^*)x^2 + \big(-2y^*\cos(\theta^*) - 2b\sin(\theta^*)\big)x + \\ &ab\sin(\theta^*)\cos(\theta^*) + by^*\cos(\theta^*) \\ &\eta(x) = -2y^*\sin(\theta^*)x + 2{y^*}^2\cos(\theta^*) + by^*\sin(\theta^*) - \\ &ab\sin^2(\theta^*) \end{aligned} \tag{20}$$

It should be noted that ζ represents the manifold in Eq.(10). Therefore, the EP of zero dynamic is equal to the EP of the robot. Now in the following proposition, we prove that the zero dynamics is stable but not asymptoticly stable.

Proposition 1. *Consider a system such as (19) in the form $\ddot{x} + \phi(x) = 0$, then its equilibrium point is stable.*

Proof. There are various ways to prove this. For example in [24] the general form of second order systems is analyzed. In this paper we use Lyapunov theorem to prove it. Consider the Lyapunov function candidate

$$V(x) = \frac{1}{2}\dot{x}^2 + \int_{x^*}^{x} \phi(x)dx \tag{21}$$

in which, $\phi(x)$ is an ascending function since $\frac{d\phi(x)}{dx}$ is positive in the workspace of the robot. Thus V is positive definite. The overall form of $\phi(x)$ and two level sets of $V(x)$ are shown in Fig.4. $\phi(x)$ is the ratio of two polynomials, nominator and denominator are second and first order polynomials respectively. The root of denominator is always between the roots of nominator, that's the reason why derivation of $\phi(x)$ is positive. As shown in Fig.4(b), because of the nonlinear form of $\int_{x^*}^{x} \phi(x)dx$ the level set of Lyapunov function are not necessarily ellipsoid.

Time derivative of (21) is as follows:

$$\dot{V} = \dot{x}\ddot{x} + \phi(x)\dot{x} = \dot{x}\big(\ddot{x} + \phi(x)\big) = 0 \tag{22}$$

Thus the EP is stable and V remains in a level set. □

The simplest idea for x^* to become asymptotically stable is to switch to a linear controller like LQR which was initially proposed by Spong in [5]. In contrast to some underactuated benchmark problems, our linearized system is not controllable about $\mathbf{X}^*$. Thus a new method is required to be developed to asymptotically stabilize the robot about its EP.

Eq.(19) is similar to a nonlinear mass-spring system which can be asymptotically stabilized by adding a damper. Like a pendulum without friction which can be asymptotically stabilized about downward EP by adding a damping term, in this paper, usage of a damping term is proposed in the controller structure. This term is similar to friction, but it is not physical and is created by the control law. However, because of the strong input coupling in parallel robots, it is not possible to add a term only to a subsystem of the robot. Therefore, the control law should be redesigned.

Theorem. *Consider the underactuated cable-driven robot with dynamics of Eq.(7) and following control law*

$$\boldsymbol{\tau} = \begin{bmatrix} \frac{x-a\cos(\theta)}{l_1} & \frac{y-a\sin(\theta)}{l_1} \\ \frac{x-b+a\cos(\theta)}{l_2} & \frac{y+a\sin(\theta)}{l_2} \end{bmatrix}^{\boldsymbol{-T}} \begin{bmatrix} -\gamma\dot{x} \\ 0 \end{bmatrix} + \boldsymbol{\tau_{new}} \tag{23}$$

where input $\boldsymbol{\tau}_{new}$ is equal to control law (11) and (15), with $\mathbf{K}, \mathbf{\Gamma}, \gamma > 0$ and

$$\begin{aligned} &k' = |f(\mathbf{X}, \dot{\mathbf{X}})\gamma\dot{x}| = |\mathbf{J}_3^T \begin{bmatrix} \frac{x-a\cos(\theta)}{l_1} & \frac{y-a\sin(\theta)}{l_1} \\ \frac{x-b+a\cos(\theta)}{l_2} & \frac{y+a\sin(\theta)}{l_2} \end{bmatrix}^{-T} \\ &\begin{bmatrix} -\gamma\dot{x} \\ 0 \end{bmatrix}| = |\frac{2y^2\cos(\theta) - 2xy\sin(\theta) + by\sin(\theta) - ab\sin^2(\theta)}{2ax\sin(\theta) - 2ay\cos(\theta) + by - ab\sin(\theta)} \\ &\gamma\dot{x}| \end{aligned} \tag{24}$$

Then EP of the robot is asymptotically stable.

Proof. First, Substitute Eq.(23) in Eq.(7), resulting in the following dynamics:

$$\begin{aligned} m\ddot{x} + \gamma\dot{x} &= \mathbf{J}_1^T\tau_{new} \\ m\ddot{y} + mg &= \mathbf{J}_2^T\tau_{new} \\ I\ddot{\theta} - f(\mathbf{X}, \dot{\mathbf{X}}) &= \mathbf{J}_3^T\tau_{new} \end{aligned} \tag{25}$$

where $\mathbf{J}_i^T$ is ith row of $\mathbf{J}^T(X)$, the nominator of $f(\mathbf{X}, \dot{\mathbf{X}})$ is $\det[\mathbf{J_2}, \mathbf{J}_3]$ and denominator is $\det[\mathbf{J}_1, \mathbf{J}_2]$, thus on the manifold (10) the above fraction is equal to one. The dynamics of y, θ are as follows

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} (\ddot{\mathbf{X}}' - \dot{\boldsymbol{\nu}}) + \mathbf{K}\mathbf{s} + k'\text{sgn}(\mathbf{s}) + \begin{bmatrix} 0 \\ f(\mathbf{X}, \dot{\mathbf{X}})\gamma\dot{x} \end{bmatrix} = \\ \mathbf{M}'\dot{\mathbf{s}} + \mathbf{K}\mathbf{s} + k'\text{sgn}(\mathbf{s}) + \mathbf{d} = 0 \tag{26}$$

where $\boldsymbol{s}$ is given in Eq.(14). Note that by using the above control law, the dynamics of y remain unchanged. $f(\mathbf{X}, \dot{\mathbf{X}})\gamma\dot{x}$ is considered as disturbance and is rejected using $k'\text{sgn}(\mathbf{s})$.

Now consider the Lyapunov function $V = \frac{1}{2}\mathbf{s^T}\mathbf{M}'\mathbf{s}$. Derive derivation of V respect to time

$$\dot{V} = \mathbf{s^T}\mathbf{M}'\dot{\mathbf{S}} \leq -\mathbf{s^T}\mathbf{K}\mathbf{s} - k'|\mathbf{s}| - |\mathbf{d}||\mathbf{s}| = -\mathbf{s^T}\mathbf{K}\mathbf{s} - k'|\mathbf{s}| \\ - |f(\mathbf{X}, \dot{\mathbf{X}})\gamma\dot{x}||\mathbf{s}| = -\mathbf{s^T}\mathbf{K}\mathbf{s} \tag{27}$$

s converges to zero, and therefore, $\tilde{y}, \tilde{\theta}$ converge to zero. Furthermore, dynamics of x is equal to (19) plus the damping term $\gamma\dot{x}$. Therefore x also converges to x^*. □

Note that control law (23) was derived by multiplying inverse of $[\mathbf{J}_1, \mathbf{J}_2]$ and damping term $\gamma\dot{x}$ by x dynamics.

Remark 1. *The control law is chattering free, because the gain of sgn is proportional to $\dot{x}$ which will vanish as x converges to its equilibrium point.*

Remark 2. *Since internal dynamics of the robot is ISS with respect to x, we conclude that zero dynamic is asymptotically stable. This yields to asymptotic stability of the internal dynamics.*

Remark 3. *Positive tension is guaranteed because we suppose that the robot is in feasible workspace [25] and if one of the τ_i's is zero, the robot will descend and swing around the other anchor point.*

V. Simulation Results

This section is devoted to simulation of the proposed control method on the underactuated cable robot in mind. The physical parameters of the robot are set to:

$$a = 0.25m \quad b = 7.9m \quad m = 2Kg \quad I = 0.04Kgm^2.$$

Initial and desired values of the robot are also set to:

$$\begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -0.5 \end{bmatrix}, \quad \begin{bmatrix} x^* \\ y^* \\ \theta^* \end{bmatrix} = \begin{bmatrix} 5.924 \\ -3 \\ 0.5 \end{bmatrix}.$$

Notice that the desired point satisfies equation (10) whereas initial point is selected freely. The controller parameters are set to:

$$\boldsymbol{\Gamma} = 0.5\boldsymbol{I}, \qquad \boldsymbol{K} = \boldsymbol{I}, \qquad \gamma = 0.8,$$

where $\boldsymbol{I} \in \mathbb{R}^{2\times 2}$ denotes the identity matrix. As shown in figure 5, x is plotted by blue-dotted and y, θ are shown by green dash-dot and solid red lines, respectively. As shown in Fig.5(a), without damping injection ($\gamma = 0$) y, θ converge to y^*, θ^* and x is stable about x^*. By damping injection to x dynamics, all of configuration variables converge to their desired values as shown in figures 5(b) and 5(c). Convergence time is about 25 seconds and can be reduced by increasing the controller gain $\boldsymbol{K}, \boldsymbol{\Gamma}$, however, this leads to larger control signals. The reason for swing-like movements in transient response of θ and x is the term $f(\mathbf{X}, \dot{\mathbf{X}})$ in zero dynamics. Therefore, these swings lead to oscillatory behavior of inputs as shown in Fig. 5(d). However as already discussed the control law is chattering free because the sgn term vanishes when error converges to zero.

VI. Conclusions and Future Work

In this paper, regulator design of an underactuated cable-driven robot is proposed. The kinematic and dynamic models of the robot are derived, and the positions in which the robot can stay with constant inputs are exploited. By analyzing zero dynamics of the robot, it is shown that the equilibrium point is just stable and may have an oscillatory behavior. By composition of sliding mode control and damping injection in IDA-PBC method, asymptotic stability of desired position is ensured by Lyapunov theorem. Furthermore it is shown that by this structure the control signals are chattering free.

Our future research is focused on generalizing this approach for a range of underactuated robots, passivity-based analysis of the designed controller, transforming the equations to port-Hamiltonian structure, and Considering kinematic and dynamic uncertainties and developing robust and adaptive controllers.

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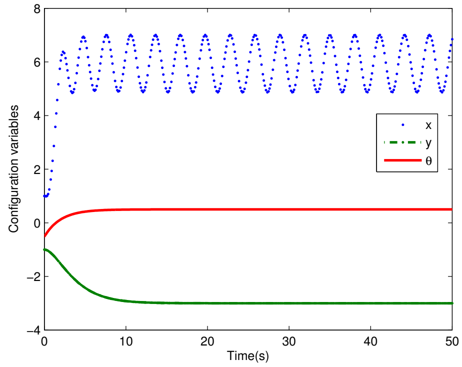
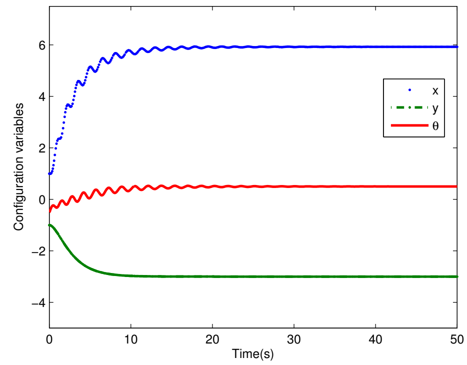
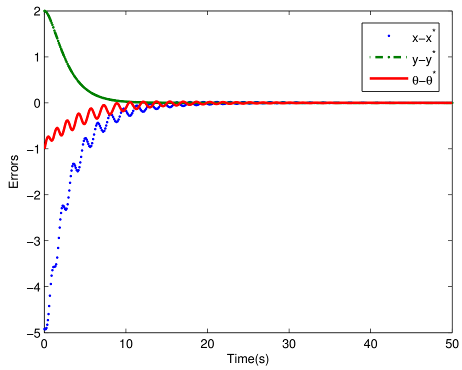


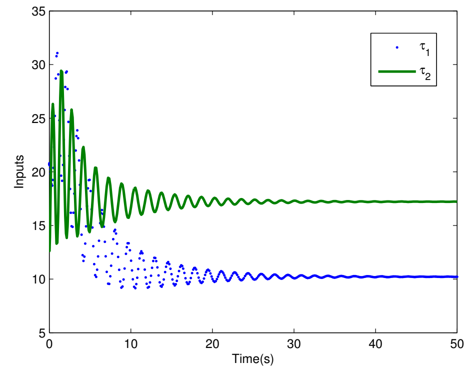
(a) Configuration variables of the robot using control law (11) and (15).



(b) Configuration variables of the robot using control law (11), (15) and (23).



(c) Configuration error variables of the robot using control law (11), (15) and (23).



(d) Cables forces with the proposed control law.

Fig. 5. Simulation results with the proposed controller. It's clear that without damping injection, x has oscillatory behavior. By adding a damping term to x dynamics, all of configuration variables converge to their desired values.

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