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A switched SDRE filter for state of charge estimation of lithium-ion batteries

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ABSTRACT

Lithium-ion (Li-ion) batteries need very precise monitor of the state of charge (SOC) to ensure a long cycle life. Hence, a knowledge of the SOC is important for Li-ion batteries. Although SOC cannot be measured directly, it can be estimated from direct measurement variables based on a model of the battery. Single-Particle-Model (SPM), a reduced-order nonlinear electrochemical model, is commonly used for this purpose. State-dependent-Riccati-equation (SDRE) filter is chosen as the estimator due to its high-flexibility in handling the model's nonlinearity. However, performance of this filter is limited in presence of uncertainties. To tackle this problem, in this paper, a switching concept is induced into SDRE filter, in the form of switched estimation error covariance matrix with a certain frequency. Thus, by changing the Riccati equation dynamic in SDRE filter and proper adjustment of estimation error covariance matrix eigenvalues, performance and robustness of the common SDRE filter is significantly improved for Li-ion SOC estimation. To analyze the fidelity of such a filter in further applications, stability analysis is carried out on a class of nonlinear systems, and ultimate bound of estimation error is analytically obtained, and the influence of switching is investigated. Simulation results reveal effectiveness of the proposed filter compared to common SDRE filter, extended Kalman filter and variable structure approaches. Furthermore, experimental results verify the effectiveness of the proposed method in practice.

1. Introduction

Batteries are the vital energy conservation elements used in electric vehicles. Among various battery types, the most common is Li-ion, owing to its high energy density [1]. Li-ion batteries are, however, highly sensitive to their SOC; Over-discharge reduces their rechargeable capacity and over-charging can lead to serious damages or even explosion [2]. Thus, SOC is considered essential in both maintenance and safety contexts. Besides, SOC monitoring keeps the driver informed of available energy for the remaining travel distance.

SOC is not directly measurable, hence, estimation and evolved filter hypotheses are exerted in this regard [3]. In these methods, SOC is estimated through a mathematical model based on the battery terminal voltage and charge/discharge current. Plenty of models have been developed for Li-ion batteries, e.g. electrochemical models [4,5]; and equivalent circuit model (ECM), among which ECM is more prevalent due to ease of implementation [6–8]. ECM models, however, are not of sufficient accuracy, due to merely modeling the dominant dynamics and not the electro-chemicals. In this article, a reduced order electrochemical model, called single-particle model [9], is used, which yields a non-linear model of SOC in terms of battery terminal voltage.

To deal with the model nonlinearity, a non-linear filter is exploited. Several recognized methods to design non-linear filters are particle filter [10], variable structure, high-order sliding-mode [11], designing methods based on the Lyapunov theorem, recursive Bayesian approach [12], Extended Kalman Filter (EKF), and designing methods based on state dependent Riccati equation (SDRE) [13]. SDRE method, unlike EKF, tackles the nonlinearity by parameterizations and not by linearization and elimination of non-linear effects. Furthermore, in comparison with variable structure filter, SDRE filter estimation is more smooth, and it has less fluctuation. Moreover, to have a better intuition on SDRE performance a comprehensive comparison considering multiple filters is reported in the simulation part. SDRE method's high degree of freedom contributes to singularity avoidance and unobservable regions. Despite its advantages, model uncertainty causes limitations in filter implementation. The model uncertainties cause an ultimate error bound in estimating the SOC of the battery. Hence, the filter, used in practice, should be robust to changes in the parameters of the battery model. Some nominated researches in this area are as follows. In [14], a recursive total least squares algorithm is utilized along

Abbreviations: SOC, State of charge; Li-ion, Lithium-ion; SDRE, State dependent Riccati equation; SDDRE, State dependent differential Riccati equation; EKF, Extended Kalman filter; ECM, Equivalent circuit model; SDC, State dependent coefficient

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with an SOC observer, to eliminate the bias in the identified parameters of the model as a result of noise in sensors; which also increases the accuracy in SOC and parameter estimation. A comparative study is made on model-based capacity estimation algorithms for Li-ion batteries under an accelerated aging test [15]. Further, in [16], a multi-gain-switching approach is proposed which handles sensor drifts and modeling mismatches. In [17] to improve the estimation accuracy and robustness, a Bayesian dual-filtering framework is employed which estimates parameters and states.

In this article, and as another means for robust estimation, time-dependent switching-based SDRE filter is proposed, to increase the structural robustness against model uncertainty. In other words, combination of switching concept and traditional SDRE filter enhances the SDRE filter performance in presence of uncertainties. Despite the fact that switching concept is not commonly reported in robust filter design, the method is widespread in the dual problem, i.e., robust controller design. In [18], in comparison with pulse-width modulation based controllers, switched linear equations are employed in the controller design procedure. Thus, the suggested controller results faster transient responses. In [19] an economic dispatch problem of renewable hybrid power systems is modeled as an optimal control problem of switched dynamic systems. Finally, in [20], a switched consensus-based distributed controller is presented for each load bus in transmission networks.

It is noteworthy that in this method switching concept is realized through using switched estimation error covariance matrix with a certain switching frequency determined in filter equations and the main result of this approach is the adjustment of estimation error covariance matrix eigenvalues which yields to robustness and lower amount of ultimate error bound. Given the application of the proposed filter in estimation theory, stability analysis based on Lyapunov theorem is elaborated first in this paper. Then robustness analysis is performed and the impact of switching on the ultimate bound of estimation error is verified. Various simulations are given to ascertain effectiveness of the proposed filter, and finally, real-time implementation results of the filter is given to show proof of theoretical development concept in our application.

The rest of this paper is organized as follows: In Section 2 the mathematical preliminaries are addressed. In Section 3 sufficient conditions for stability of the filter estimation error are introduced. Section 4 focuses on the determination of estimation error ultimate bound. Simulation and experimental results are given in Section 5. Finally, concluding remarks are given in Section 6.

2. Proposed filter and preliminaries

Consider the following general representation of a nonlinear system.

$$\dot{x}(t) = f(x, u) + \Delta f(x, u) + Gw_0(t) \tag{1}$$

$$y(t) = h(x, u) + \Delta h(x, u) + D_1 v_0(t) \tag{2}$$

where $x(t) \in R^n$ denotes the system state vector, $u(t) \in R^m$ is the input, $y(t) \in R^p$ represents the measured output, $w_0(t)$ and $v_0(t)$, which stand for exogenous disturbance inputs, denote process and measurement noises with unknown statistical properties, respectively, and $\Delta f(x, u)$ and $\Delta h(x, u)$ are the model uncertainties.

To use SDRE approach, the equations are rewritten in state dependent coefficient (SDC) form. Referring to [21], most non-linear state equations are transformable to SDC form. Hence, system dynamics may be represented by:

$$\dot{x}(t) = A(x)x + \Delta f(x) + B(x)u + \Delta B(x)u + Gw_0(t) \tag{3}$$

$$y(t) = C(x)x + \Delta h(x) + D(x)u + \Delta D(x)u + D_1 v_0(t) \tag{4}$$

where it is assumed that $\Delta f(x, u) = \Delta f(x) + \Delta B(x)u$ and $\Delta h(x, u) = \Delta h(x) + \Delta D(x)u$.

Remark 1. In general, SDC form is not unique for multivariable systems.

Remark 2. If $A_1(x)$ and $A_2(x)$ are two separate factors coefficients of $f(x)$, then, $A_3(x) = M(x)A_1(x) + (I - M(x))A_2(x)$ is also a parameterizations of $f(x)$ for any matrix function $M(x) \in R^{n\times n}$. This holds also for $B(x)$, $C(x)$ and $D(x)$.

Remark 2 is a unique characteristic of SDRE approach, which avoids singular and unobservable points of the system, resulting in increased efficacy in design process. Moreover, it is beneficial in fulfilling the Lipschitz condition, addressed later. The aim is to design a filter which robustly estimates the state vector $x(t)$, using measurable output vector, $y(t)$. Consider the proposed filter structure as

$$\dot{\hat{x}}(t) = A(\hat{x})\hat{x}(t) + B(\hat{x})u(t) + K(\hat{x}, t)[y(t) - C(\hat{x})\hat{x}(t) - D(\hat{x})u(t)] \tag{5}$$

where $\hat{x}(t)$ indicates estimated state variable vector and the filter gain $K(\hat{x}, t) \in R^{n\times p}$, is defined as

$$K(\hat{x}, t) = P(t)C^T(\hat{x})R^{-1} \tag{6}$$

in which, $P(t) \in R^{n\times n}$ is a symmetric matrix, which satisfies the state dependent differential Riccati equation (SDDRE) (7) with positive definite matrices $Q \in R^{n\times n}$ and $R \in R^{p\times p}$ [22].

$$\dot{P}(t) = A(\hat{x})P(t) - P(t)C^T(\hat{x})R^{-1}C(\hat{x})P(t) + P(t)A^T(\hat{x}) + GQG^T \tag{7}$$

Consider the estimation error as

$$e(t) = x(t) - \hat{x}(t) \tag{8}$$

Subtract (5) from (3) and simplify the error dynamics to reach to:

$$\begin{aligned}\dot{e}(t) = \dot{x} - \dot{\hat{x}} &= [A(\hat{x}) - KC(\hat{x})]e(t) + \alpha(x, \hat{x}, u)\\ &- K\beta(x, \hat{x}, u) - Kv(t) + \Gamma w(t)\end{aligned} \tag{9}$$

where

$$\alpha(x, \hat{x}, u) = [A(x) - A(\hat{x})]x + [B(x) - B(\hat{x})]u \tag{10}$$

$$\beta(x, \hat{x}, u) = [C(x) - C(\hat{x})]x + [D(x) - D(\hat{x})]u \tag{11}$$

and

$$\begin{aligned}w &= (\Delta f(x) \quad \Delta B(x)u \quad w_0)^T,\\ \Gamma &= (I \quad I \quad G),\\ v &= (I \quad I \quad I)(\Delta h(x) \quad \Delta D(x)u \quad D_1 v_0)^T.\end{aligned}$$

The last step to develop the proposed filter is to include the switching concept. The switching function may be merged into $P(t)$, and the switching frequency is obtained through estimation error stability and ultimate error bound analysis. Therefore, the filter splits into a family of subsystems, that can be analyzed in switched systems domain.

3. Stability analysis

In this section, sufficient conditions to ensure the stability of estimation error are presented. To have an insight into the effect of switching, the ultimate bound of estimation error is obtained. Moreover, stability analysis is carried out based on Lyapunov method. As mentioned earlier, a family of subsystems is acquired by switching with a determined frequency. Switching approach used in this article is time-dependent and dwell-time theorem may be used to analyze the stability.

Theorem 1. Consider the non-linear continuous-time system represented in Eqs. (3) and (4), along with Eqs. ()()()(5)–(7), related to recommended filter, and assume:

1) The state dependent matrix $C(x)$ is upper-bounded by

$$\|C(x)\| \leqslant \bar{c} \tag{12}$$

where $\bar{c} > 0$ is a real number.

2) Consider that the state and input are bounded by the bounds $\sigma, \rho > 0$ for all times $t \geqslant 0$

$$\|x(t)\| \leqslant \sigma, \ \|u(t)\| \leqslant \rho \tag{13}$$

3) The solution, $P(t)$, of Riccati differential equation is bounded by

$$\underline{p}I \leqslant P(t) \leqslant \bar{p}I \tag{14}$$

where $\bar{p}$, $\underline{p}$ are positive real numbers.

4) The SDC parameterization is chosen such that matrices $A(x)$, $B(x)$, $C(x)$, $D(x)$ are at least locally Lipschitz, i.e., there exist constants k_A, k_B, k_C, $k_D > 0$ such that

$$\|A(x_1) - A(x_2)\| \leqslant k_A\|x_1 - x_2\| \tag{15}$$

$$\|B(x_1) - B(x_2)\| \leqslant k_B\|x_1 - x_2\| \tag{16}$$

$$\|C(x_1) - C(x_2)\| \leqslant k_C\|x_1 - x_2\| \tag{17}$$

$$\|D(x_1) - D(x_2)\| \leqslant k_D\|x_1 - x_2\| \tag{18}$$

for any $x_1, x_2 \in R^n$ with $\|x_1 - x_2\| \leqslant \varepsilon_A$ and $\|x_1 - x_2\| \leqslant \varepsilon_B$ and $\|x_1 - x_2\| \leqslant \varepsilon_C$ and $\|x_1 - x_2\| \leqslant \varepsilon_D$, respectively.

Then, the sufficient condition for the stability of the estimation error dynamics (9), is

$$2\kappa + 2\lambda_0\lambda_{max}^{\Pi(t)} \leqslant \lambda_{min}^{\Pi(t)\Gamma Q\Gamma^T\Pi(t)} + \lambda_{min}^{C^T(\hat{x})R^{-1}C(\hat{x})} \tag{19}$$

where κ and λ_0 are positive numbers, λ_0 is determined according to the switching frequency and $\Pi(t) = P^{-1}(t)$.

Remark 3. For any arbitrary square matrix A, λ_{max}^A and λ_{min}^A denotes the maximum and minimum eigenvalues of A, respectively.

Before proving the sufficient condition of stability, presented in Eq. (19), it is necessary to point out the assumptions of the proposition. These are represented in the form of 3 remarks. Then the proof of the theorem is exposed.

Remark 4. The second assumption is not a strict condition, because, the state variables representing physical quantities, are often bounded. Moreover, bounded control input is quite obvious. Therefore, inequalities in (13) are easily verified.

Remark 5. The inequality (14), the key factor in stability analysis, is closely related to observability and delectability characteristics of the observed system. This is related to the boundary of the Riccati equation solution, based on the following three conditions [23]:

1- The designing matrix Q is positive definite and matrix $A(x)$ has limited norm.

2- The SDC form is chosen in a way that $\{A(x), C(x)\}$ is uniformly detectable according to Definition 1.

3- The initial condition $P(0)$ in (7) is positive definite.

Definition 1. $\{A(x), C(x)\}$ is called a parametrization of uniformly detectable SDC of the system if a bounded matrix, $\Lambda(x)$, and a real number, $\gamma > 0$, exist such that the following inequality is satisfied for any $\omega, x \in R^n$:

$$\omega^T[A(x) + \Lambda(x)C(x)]\omega \leqslant -\gamma\|\omega\| \tag{20}$$

Remark 6. The Lipschitz condition in the fourth assumption is commonly exerted to SDRE problems [24], and is not to be considered as unprecedented limiting condition.

Proof. The following lemma is needed to complete the proof of the theorem in two steps.

Lemma 1. Consider a family of subsystems introduced by (9). Assume Lyapunov candidate function V_p for subsystem p and two class-K_∞ functions α_1 and α_2 and a real positive constant λ_0 such that

$$\alpha_1(|x|) \leqslant V_p(x) \leqslant \alpha_2(|x|) \quad \forall\, x, \quad \forall\, p \in \varrho \tag{21}$$

$$\frac{\partial V_p(x)}{\partial x} f_p(x) \leqslant -2\lambda_0 V_p(x) \quad \forall\, x, \quad \forall\, p \in \varrho \tag{22}$$

$$V_p(x) \leqslant \mu V_q(x) \quad \forall\, x, \quad \forall\, p, q \in \varrho \tag{23}$$

where ϱ is the set of subsystems. Then, the non-linear continuous switched system (9) is stable for every switching signal with average dwell-time τ_a if

$$\tau_a > \frac{\log\mu}{2\lambda_0} \tag{24}$$

where μ is a positive constant.

For the proof of Lemma 1 refer to [25]. Note that inequality (21) is valid due to Assumption 3. Hereafter, establishment of (22) and (23) is studied in two steps.

Step 1: The sufficient condition to establish (22) is obtained analytically. In this regard, the following preparation is needed.

Lemma 2. Consider the positive definite matrix R with $p \times p$ dimension and assume that we have $R \geqslant \underline{r}I$. Further, consider matrix $K(\hat{x}, t)$ and non-linear terms $\alpha(x, \hat{x}, u)$ and $\beta(x, \hat{x}, u)$ defined by (10) and (11). Then, according to the mentioned four assumptions, the real numbers $\varepsilon, \kappa > 0$ exist such that matrix $\Pi(t) = P^{-1}(t)$ satisfies the following inequality for any $\|(x - \hat{x})\| \leqslant \varepsilon$:

$$e^T(t)\Pi(t)[\alpha - K\beta] \leqslant \kappa\|e(t)\|^2 \tag{25}$$

where following holds for $\|e(t)\| \leqslant \min(\varepsilon_A, \varepsilon_B, \varepsilon_C, \varepsilon_D)$:

$$\kappa = \frac{(k_A\sigma + k_B\rho)}{\underline{p}} + \frac{\bar{c}}{\underline{r}}\left(k_C\sigma + k_D\rho\right) \tag{26}$$

Proof. Applying triangle inequality and using equations $K(\hat{x}, t) = P(t)C(\hat{x})^T R^{-1}$ and $\Pi P = I$ by considering the relations (12)–(18), yields to the following inequalities:

$$\|\alpha(x, \hat{x}, u)\| \leqslant \|[A(x) - A(\hat{x})]x\| + \|[B(x) - B(\hat{x})]u\| \\ \leqslant (k_A\sigma + k_B\rho)\|x - \hat{x}\| \tag{27}$$

$$\|\beta(x, \hat{x}, u)\| \leqslant \|[C(x) - C(\hat{x})]x\| + \|[D(x) - D(\hat{x})]u\| \\ \leqslant (k_C\sigma + k_D\rho)\|x - \hat{x}\| \tag{28}$$

then exploiting $\left\|\Pi\right\| \leqslant \frac{1}{\underline{p}}$ and $\left\|R^{-1}\right\| \leqslant \frac{1}{\underline{r}}$ for $\|x - \hat{x}\| \leqslant \varepsilon$ with $\varepsilon = \min($ $\varepsilon_A, \varepsilon_B, \varepsilon_C, \varepsilon_D)$, yields to

$$\left\|e^T(t)\Pi(t)\left(\alpha - K\beta\right)\right\| \leqslant \left\|x - \hat{x}\right\|\left\|\frac{(k_A\sigma + k_B\rho)}{\underline{p}}\right\|\left\|x - \hat{x}\right\| \\ + \left\|x - \hat{x}\right\|\left\|\frac{\bar{c}}{\underline{r}}\left(k_C\sigma + k_D\rho\right)\right\|\left\|x - \hat{x}\right\| \tag{29}$$

Hence, considering (26) the inequality (25) is immediately concluded. □

Stability analysis of switched systems using direct method of Lyapunov theorem may be accomplished by selection of merely one Lyapunov function for all subsystems, or distinct Lyapunov functions for each subsystem. Since the former is slightly conservative, analysis is performed using the latter. To this end, the following Lyapunov functions are defined for subsystems and the derivatives are obtained and analyzed.

$$V(e(t), t) = e^T(t)\Pi(t)e(t) \tag{30}$$

where $\Pi(t) = P^{-1}(t)$ and $\dot{\Pi} = -\Pi\dot{P}\Pi$, also considering the establishment of (14), the following holds

$$\frac{1}{\bar{p}}\left\|e(t)\right\|^2 \leqslant V\left(e(t), t\right) \leqslant \frac{1}{\underline{p}}\left\|e(t)\right\|^2 \tag{31}$$

where $\underline{p}$ and $\overline{p}$ are the minimum and maximum eigenvalues of matrix $P(t)$. Eq. (31) indicates that $V(e(t), t)$ is positive definite and decrescent, and thus an appropriate Lyapunov candidate. Next, the derivatives of Lyapunov candidate along the trajectories of (9) are analyzed as follows.

$$\begin{aligned}\dot{V} &= \dot{e}^T(t)\Pi(t)e(t) + e^T(t)\dot{\Pi}(t)e(t) + e^T(t)\Pi(t)\dot{e}(t)=\\ &e^T(A-KC)^T\Pi e + e^T\Pi(A-KC)e\\ &- e^T\Pi\Gamma Q\Gamma^T\Pi e + e^TC^TR^{-1}Ce - e^T\Pi Ae - e^TA^T\Pi e\\ &+ w^T\Gamma^T\Pi e + e^T\Pi\Gamma w - v^TK^T\Pi e - e^T\Pi Kv\\ &+ (\alpha - K\beta)^T\Pi e + e^T\Pi(\alpha-K\beta)\end{aligned} \tag{32}$$

After substitution of K from Eq. (6) for some terms and some simplifications, the following result is obtained:

$$\begin{aligned}\dot{V} &= e^T(t)\Pi(t)\Gamma w(t) + w^T(t)\Gamma^T\Pi(t)e(t)\\ &+ (\alpha-K\beta)^T\Pi(t)e(t) + e^T(t)\Pi(t)(\alpha-K\beta)\\ &- e^T(t)\Pi(t)\Gamma Q\Gamma^T\Pi(t)e(t) - v^T(t)R^{-1}C(\hat{x})e(t)\\ &- e^T(t)C^T(\hat{x})R^{-1}C(\hat{x})e(t) - e^T(t)C^T(\hat{x})R^{-1}v(t)\end{aligned} \tag{33}$$

By applying Lemma 2, the above equation will be as follows:

$$\begin{aligned}\dot{V} &\leqslant -v^T(t)R^{-1}C(\hat{x})e(t) - e^T(t)C^T(\hat{x})R^{-1}v(t)\\ &- e^T(t)\Pi(t)\Gamma Q\Gamma^T\Pi(t)e(t) - e^T(t)C^T(\hat{x})R^{-1}C(\hat{x})e(t)\\ &+ e^T(t)\Pi(t)\Gamma w(t) + w^T(t)\Gamma^T\Pi(t)e(t) + 2\kappa\|e(t)\|\end{aligned} \tag{34}$$

hence, considering the following relation for any arbitrary matrix A:

$$\lambda_{min}^A\|e(t)\|^2 \leqslant e^T(t)Ae(t) \leqslant \lambda_{max}^A\|e(t)\|^2 \tag{35}$$

yields

$$\dot{V} \leqslant (2\kappa - (\lambda_{min}^I + \lambda_{min}^{II}))\|e(t)\|^2 + \|2w^T(t)\Gamma^T\Pi(t) - 2v^T(t)R^{-1}C(\hat{x})\| \, \|e(t)\| \tag{36}$$

where $\lambda_{min}^I = \lambda_{min}^{\Pi(t)\Gamma Q\Gamma^T\Pi(t)}$ and $\lambda_{min}^{II} = \lambda_{min}^{C^T(\hat{x})R^{-1}C(\hat{x})}$.

Obviously, (36) is a quadratic function of $\|e(t)\|$ by which analysis of that, may get us to the ultimate bound of the estimation error. Now if

$$2\kappa < \lambda_{min}^I + \lambda_{min}^{II} \tag{37}$$

then the obtained quadratic function in (36), attains its maximum ε at $\left\|e(t)\right\| = -\frac{\eta^2}{2[2\kappa - (\lambda_{min}^I + \lambda_{min}^{II})]}$ such that

$$\varepsilon = -\frac{(\eta^2)^2}{4[2\kappa - (\lambda_{min}^I + \lambda_{min}^{II})]} \tag{38}$$

where η^2 indicates the amount of uncertainty in the model. This clarifies the existence of ultimate bound for the estimation error. Rewrite (36) as

$$\dot{V}(e(t), t) \leqslant -\alpha_3(\|e(t)\|) + \varepsilon \tag{39}$$

where $\alpha_3(\|e(t)\|)$ is a positive definite function. Considering (31), the sufficient condition for (22), is apparently equivalent to (19).

Step2: Assume that Lyapunov function for subsystem p is $V_1 = e_1^T\Pi_0(t)e_1$ and $V_2 = e_2^T\Pi(t)e_2$, at times t_1 and t_2. Having $\Pi(t) > \Pi_0(t)$ and taking (19), relation $e_1 > e_2$ is surely verified. Now, if switching happens at time t_3, the Lyapunov function would be $V_3 = e_3^T\Pi_0(t)e_3$, hence using the reality $e_1 > e_2 > e_3$, (23) is proved as well. □

Remark 7. If (14) holds and Π_0 is the initial condition for $\Pi(t)$, Then, $\Pi(t) > \Pi_0(t)$ is a rational assumption for all $t > 0$ before the switching happens.

Remark 8. The switching frequency and the sufficient condition for stability are interrelated through (24).

Remark 9. We claim that the switching on the matrix $P(t)$ is an effective tool to maximize $\lambda_{min}^{\Pi(t)}$ and minimize $\lambda_{max}^{\Pi(t)}$; since, it induces (19) to hold for higher bound of input and state variables according to (26). Although high frequencies of switching seem to yield better results, a compromise is required between switching frequency and λ_0, according to Eq. (24). Since the switching frequency is directly used in (19) and problems could arise by its excessive increase, it must be determined in an appropriate interval.

4. Ultimate bound analysis of the estimation error

In this section, the uncertainty amount which was formerly displayed as η^2 is expressed in parameters. Likewise, the ultimate bound of the estimation error for this uncertainty is obtained analytically.

Theorem 2. According to (36) and (39) consider η^2 as

$$\eta^2 = \left\|2w^T(t)\Gamma^T\Pi(t) - 2v^T(t)R^{-1}C(\hat{x})\right\| = \left\|\psi\right\|(\lambda_{max}^F)^{\frac{1}{2}} \tag{40}$$

where matrices ψ and F are defined as

$$\psi = (w^T \quad v^T)^T \tag{41}$$

$$F = 4\begin{pmatrix} \Gamma^T\Pi^2\Gamma & -\Gamma^T\Pi C^T(\hat{x})R^{-1} \\ -(\Gamma^T\Pi C^T(\hat{x})R^{-1})^T & R^{-1}C(\hat{x})C^T(\hat{x})R^{-1}\end{pmatrix} \tag{42}$$

If the sufficient condition (19) holds, then error ultimate bound will be:

$$\frac{\eta^2}{\lambda_{min}^I + \lambda_{min}^{II} - 2\kappa}\left(\frac{\overline{p}}{\underline{p}}\right)^{\frac{1}{2}} \tag{43}$$

Proof. if (19) holds, referring to (39) the term $\alpha_3(\|e(t)\|)$ will be positive definite and error ultimate bound results from ε. Consequently, to find the ultimate bound of estimation error, choosing $\varepsilon = \alpha_3(\|e(t)\|)$ and using the following equation from (36)

$$\alpha_3(\|e(t)\|) = -(2\kappa - (\lambda_{min}^I + \lambda_{min}^{II}))\|e(t)\|^2 - \eta^2\|e(t)\| + \varepsilon \tag{44}$$

yield to,

$$\alpha_3^{-1}(\varepsilon) = -\frac{\eta^2}{2\kappa - (\lambda_{min}^I + \lambda_{min}^{II})} \tag{45}$$

Furthermore, $\forall\, t \geqslant T + t_0$ inequality (46) is verified [26].

$$\|e(t)\| \leqslant \alpha_1^{-1}(\alpha_2(\zeta)) \tag{46}$$

where ζ is obtained from (45) and the two class-KL functions, α_1 and α_2 are obtained from (31). Using (46), the error ultimate bound is given in the form of (43). □

Remark 10. The amount of uncertainty η^2 is obtained with analysis of $\|e(t)\|$ coefficient in (36).

Remark 11. Besides controlling the boundaries of $P(t)$ (or $\Pi(t)$) eigenvalues, as mentioned in Remark 9, switching alleviates the error bound in (43), by increasing λ_{min}^I, and decreasing κ and also $\left(\frac{\overline{p}}{\underline{p}}\right)^{\frac{1}{2}}$, and therefore, is very effective.

5. Simulation and experimental results

5.1. Simulation studies

In this section, simulation results of the proposed filter are presented in MATLAB and the filter performance is verified in presence of model uncertainties. As mentioned earlier the objective is to estimate the Li-ion battery SOC via measurement of battery terminal voltage. In this regard, a reduced order electrochemical model of Li-ion battery, called SPM, is used, which models electrochemical phenomena besides the dominant physical relations [9].

SPM is based on approximation of the electrodes as spherical particles along with volume-averaging assumptions which leads to two following linear PDEs:

$$\frac{\partial c_s^\pm}{\partial t} = \frac{D_s^\pm}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c_s^\pm}{\partial r}\right)$$
$$\left.\frac{\partial c_s^\pm}{\partial r}\right|_{r=0} = 0,\ \left.\frac{\partial c_s^\pm}{\partial r}\right|_{r=R^\pm} = \frac{\pm I}{a_s^\pm FD_s^\pm AL^\pm} \tag{47}$$

where $c_s^\pm$ is the Li-ion concentration of the positive and negative electrode and I is the charge/discharge current.

Assuming the positive electrode Li-ion concentration is approximated as a function of the negative one and conservation of the number of Li-ions in the cell [27] leads to observable forms for Li-concentration states when the negative electrode diffusion PDE is discretized using the method of lines technique (based on finite central difference method). Next, considering spatial domain is discretized into five nodes, where $[C_{S0}, C_{S1}, C_{S2}, C_{S3}, C_{S4}]$ are the Li-ion concentration at the nodes (Fig. 1a) yields to the following linear state space:

$$\dot{C}_{S1} = -2aC_{S1} + 2aC_{S2}$$
$$\dot{C}_{S2} = (\tfrac{1}{2})aC_{S1} - 2aC_{S2} + (\tfrac{3}{2})aC_{S3}$$
$$\dot{C}_{S3} = (\tfrac{2}{3})aC_{S2} - 2aC_{S3} + (\tfrac{4}{3})aC_{S4}$$
$$\dot{C}_{S4} = (\tfrac{3}{4})aC_{S3} - (\tfrac{3}{4})aC_{S4} - (\tfrac{5}{4})bI \tag{48}$$

where, I is charge/discharge current, and the parameters a and b are defined as $a = \frac{D_s^-}{\Delta^2}$ and $b = \frac{1}{a_s^- F\Delta AL^-}$ in which, $\Delta = \frac{R}{M}$ is the discretization step. SPM diagram is presented in Fig. 1a where, $M+1$ is the number of considered nodes. Moreover, the last variable state C_{S4} is representing the SOC of the battery[1].

Remark 12. C_{S0} is not considered in (48) because, it does not affect any of the state equations and considering its dynamic, will make the state space unobservable.

The battery terminal voltage in terms of state variables may be expressed by the following nonlinear equation.

$$V = \frac{\bar{R}T}{a^+F}sinh^{-1}\left(\frac{I}{2a_s^+AL^+i_0^+}\right) - \frac{\bar{R}T}{a^-F}sinh^{-1}\left(\frac{I}{2a_s^-AL^-i_0^-}\right)$$
$$+ U^+(k_1C_{S4} + k_2) - U^-(C_{S4}) - R_fI \tag{49}$$

Model parameters and their values used in simulations are given in Table 1 as proposed by [9]. To have a better intuition on the nonlinear relation, represented in (49) between the battery terminal voltage and its SOC, Fig. 1b is illustrated. Clearly, the terminal voltage is a nonlinear function of lithium concentration at the last node (final state variable). This justifies usage of a nonlinear filter.

Remark 13. To check the assumptions of section III for SPM model, 1. State dependent term in (49), is

$$SDT = U^+(k_1C_{S4} + k_2) - U^-(C_{S4}) \tag{50}$$

in which

$$U^- = 0.124 + 2.5e^{(-70x_n)} + 0.0351\tanh(\tfrac{x_n-0.286}{0.13})$$
$$-0.055\tanh(\tfrac{x_n-0.9}{0.3}) - 0.67\tanh(\tfrac{x_n-0.99}{0.2})$$
$$-0.077\tanh(\tfrac{x_n-0.5}{0.034}) - 0.121\tanh(\tfrac{x_n-0.194}{0.142})$$
$$-0.082\tanh(\tfrac{x_n-0.98}{0.0164}) - 0.011\tanh(\tfrac{x_n-0.124}{0.0226})$$
$$-0.0055\tanh(\tfrac{x_n-0.105}{0.029}) \tag{51}$$

$$U^+ = 3.5796 + 0.51163\arctan(4.54 - 60.95x_p)$$
$$+ 0.46309\arctan(-203.36x_p + 202.43) \tag{52}$$

where $x_n = C_{S4}/C^-_{(s,max)}$ and $x_p = (k_1C_{S4} + k_2)/C^+_{(s,max)}$. To obtain $C(\hat{x})$ one can use $C(\hat{x}) = \frac{\mathrm{SDT}}{\hat{x}}$, where the SDT is given in (50). Recalling that $\hat{x}$ represents the SOC of the battery, it is obvious that there exists an upper-bound for matrix $C(\hat{x})$. Thus, the first assumption is valid.

2. Input and state variables are physical values for a stable system, and thus are bounded. Therefore, the second assumption is valid.

3. The third assumption is checked through Remark 5 and its condition (20). In this regard, it is enough to show that $A(\hat{x}) + \Lambda(\hat{x})C(\hat{x})$ is negative definite for a bounded matrix $\Lambda(\hat{x})$. We have single output besides using (50) to obtain $C(\hat{x})$. Moreover, $C(\hat{x})$ is bounded and is not null, since it represents the voltage of the battery; thus, it is invertible. Therefore, $\Lambda(\hat{x})$ may be chosen equal to $-\Theta I_n(C(\hat{x}))^{-1}$; where, Θ is a positive constant and I_n is identity matrix with corresponding dimension. According to (48) the dynamics of state variables is linear and yields to a constant matrix $A(\hat{x})$. Thus, Θ may be chosen such that relation (20) holds, for some γ.

4. Lipschitz condition for matrices $A(\hat{x})$ and $B(\hat{x})$ results immediately from linearity property of dynamics. Also, $C(\hat{x})$ is Lipschitz due to continuity. Moreover, $D(\hat{x})$ is constant with respect to $\hat{x}$, and thus Lipschitz. Therefore, assumption four is also valid.

Fig. 2 shows the flowchart of the whole algorithm. To study the efficiency of the proposed filter in presence of uncertainty, simulations are carried out, taking uncertainties as $D\_u_s^- = 10000D_s^-$ and $a\_u_s^- = 0.0001a_s^-$ and $R\_u_f = .221$ (Nominal values are reported in Table 1), and initial guess for SOC estimation as 20%. Performance of proposed robust filter is benchmarked against three filters, conventional SDRE, EKF and variable structure. It should be noted that the following equation has been used in implementing the variable structure filter.

$$\dot{\hat{x}}(t) = A(\hat{x})\hat{x}(t) + B(\hat{x})u(t) + L_1[y(t) - C(\hat{x})\hat{x}(t) - D(\hat{x})u(t)]$$
$$+ L_v sgn[y(t) - C(\hat{x})\hat{x}(t) - D(\hat{x})u(t)] \tag{53}$$

In which the term $L_1[...]$ is used to handle the nominal error and the term $L_v sgn[...]$ is due to the uncertainty in the model. Also, the following is used to implement the EKF filter.

$$H_{EKF} = \begin{pmatrix} 0 & 0 & 0 & \frac{\partial V}{C_{S4}} \end{pmatrix}$$
$$\frac{\partial V}{\partial C_{S4}} = \frac{\partial U^+}{\partial x_p}\frac{k1}{C^+_{(s,max)}} - \frac{\partial U^-}{\partial x_n}\frac{1}{C^-_{(s,max)}} \tag{54}$$

where V is the battery terminal voltage in (49). Obviously, the only state variable that appears in the output is C_{S4}. Hence, we will have just the output vector to be linearized in (54). Simulations were initially performed for a case in which the design parameters Q and R of the EKF, common SDRE and the presented filter were, according to the proposed algorithms, equal to the covariance of process and measurement noise [22]. In addition, the design parameters of variable structure filter are in accordance with [9]. Note that, all filters use the same initial guess of SOC estimation and for proposed robust filter, the amount of matrix $P(t)$ is switched to its initial value in Riccati Eq. (7) frequently to realize the switching concept.

Fig. 3 shows the results of SOC estimation for a Li-ion battery in presence of aforementioned uncertainties, using conventional SDRE, EKF, variable structure and the proposed filter. As represented in this diagram, the results of the conventional SDRE and EKF are not appropriate, as these two filters cannot perform well against uncertainties of initial conditions and model. On the other hand, the results of the proposed SDRE and the variable structure filters acceptably match the real SOC. Note that, in this simulation, R and Q parameters of the proposed filter are exactly equal to R and Q in the EKF and common SDRE filter.

Now let's look at the simulation results by tuning the R and Q parameters for the EKF, common SDRE and the proposed filter.

Note: In this case, increasing Q and properly tuning R leads to higher observer gains. Hence, dependence of the filters on the model decreases, and consequently, robustness to model uncertainty increases.

Results in Fig. 4 are achieved for charges below 40%. This is due to

[1] For further details refer to [9].

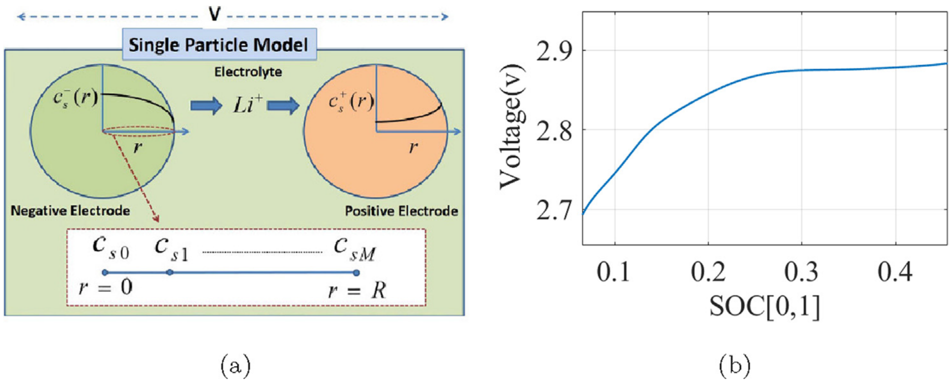


Fig. 1. (a): SPM model schematic [9]; (b): Battery terminal voltage vs. SOC in constant discharge current.

Table 1
Li-ion battery model parameters [9].

Series parameter	Series definition and unit	Series values
A	Current collector area (m^2)	0.18
$a_s^{\pm}$	Specific surface area ($\frac{m^2}{m^3}$)	3.48×10^5
$c_{s,max}^{+}$	Solid phase Li-ion saturation	22806
$c_{s,max}^{-}$	concentration ($\frac{mol}{m^3}$)	30555
$D_s^{\pm}$	Effective diffusion coefficient	5×10^{-14}
	in solid phase ($\frac{m^2}{s}$)	
$L^{\pm}$	Length of the electrodes (m)	3.4×10^{-5}
$R^{\pm}$	Radius of solid active particle (m)	5×10^{-6}
R_f	Contact film resistance (Ω)	0.222
$\varepsilon_s^{\pm}$	Active material volume fraction	0.58
i_o^{+}	Exchange current densities	0.0016
i_o^{-}		0.2
k_1	Constant	− 0.3944
k_2	Constant	20247
$\bar{R}$	Universal Gas Constant (J/mol K)	8.3144
T	Temperature (K)	298
$U^{\pm}$	Open Circuit Voltage (V)	
$a^{\pm}$	Charge Transfer Coefficient	8.3
Superscript		
±	positive/negative electrode	

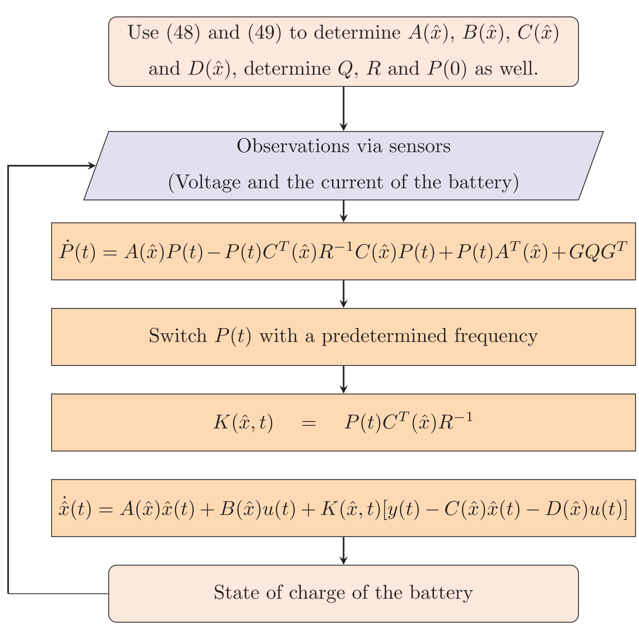


Fig. 2. The flowchart of the whole algorithm.

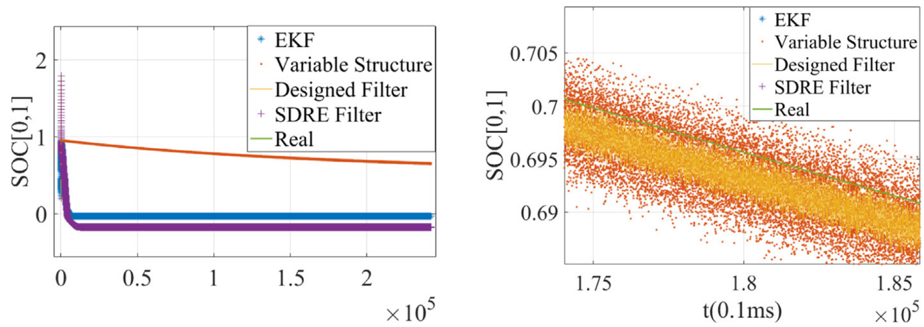
the importance of SOC estimation in this region, to stop discharging the battery and prolonging its life.

As shown in these diagrams, the common SDRE filter has less variance in SOC estimation error, however, it has more bias. EKF method yields a fairly good result, while its large variance is due to the choice of large Q matrices that have resulted in large gains. Compared to others, the variable structure approach has more variance, but less bias in its estimation error. The proposed method, as shown, improves performance of the common SDRE filter, decreases the bias of estimation error compared to common SDRE and EKF and lowers variance compared to variable structure filter. Considering that variance is more destructive than bias in estimation error, the proposed filter is comparable to the fixed-gain variable structure filter, and therefore, the common SDRE filter is empowered by switching to provide robust performance.

Next, the error ultimate bound in two cases of conventional filter and proposed filter are studied in Fig. 5, to observe the impact of switching. Eq. (43) is used in both filters to obtain estimation error bound. According to the figure, estimation error ultimate bound is much lower in proposed filter compared to conventional one. Moreover, to assert the effect of switching on maximum and minimum eigenvalues of matrix $P^{-1} = \Pi$, the two parameters are reported in Fig. 6. As depicted, maximum and minimum eigenvalues of Π are lower and higher respectively for the designed filter. Furthermore, the input discharge current is considered as it is in the pulse discharge battery test with current changes on the acceptable discharge current interval of Li-ion batteries.

To further clarify the robustness of the proposed filter, a Monte Carlo simulation is performed on 1000 different battery types, in which R_f and a_s^- along with D_s^- are the chosen parameters to be stochastic in a range from 50% to 150% of their nominal values. Note that these parameters are set to be the uncertain parameters in output model and state dynamic model of the battery, respectively. Additionally, process and measurement noise are taken into account to consider almost all the uncertainties. To have a better analysis on the performance of the presented filter, a comparative study is performed among different approaches, and the design parameters are set to be the same for the common SDRE, EKF and proposed filter.

Fig. 7a shows the mean value of SOC estimation error for each simulation. As it is seen, the performances of the designed filter and variable structure approach are close to each other and much better than the other two methods. To get a better intuition, Fig. 7a is magnified in Fig. 7b which indicates the effect of switching method on increasing robust performance of the proposed filter; as the designed filter result is significantly improved in comparison with the common SDRE due to the switching on the estimation error covariance matrix. Fig. 8a focuses on the variance of SOC estimation error of the four mentioned approaches. Obviously, the variance of variable structure estimation error is much higher than the other methods as expected due to its high



(a) Result for the entire simulation time (b) Magnified result for a specific interval

Fig. 3. Estimation results for the filters, using Q and R as covariance of process and measurement noises.

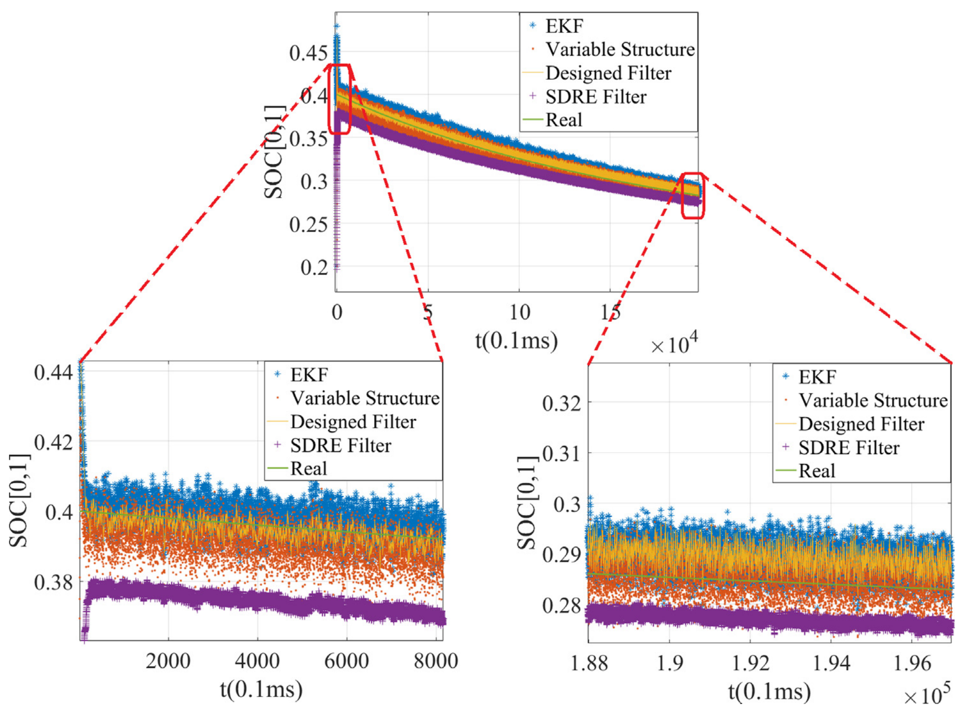


Fig. 4. Estimation results for tuned filters using large Q and proper R.

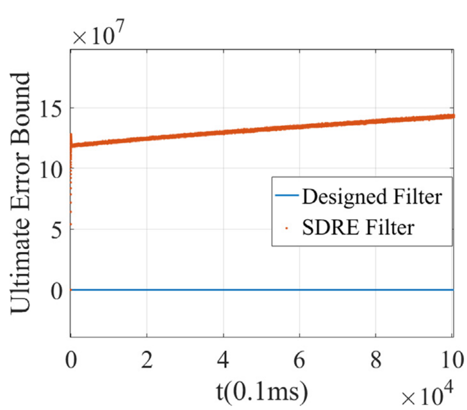
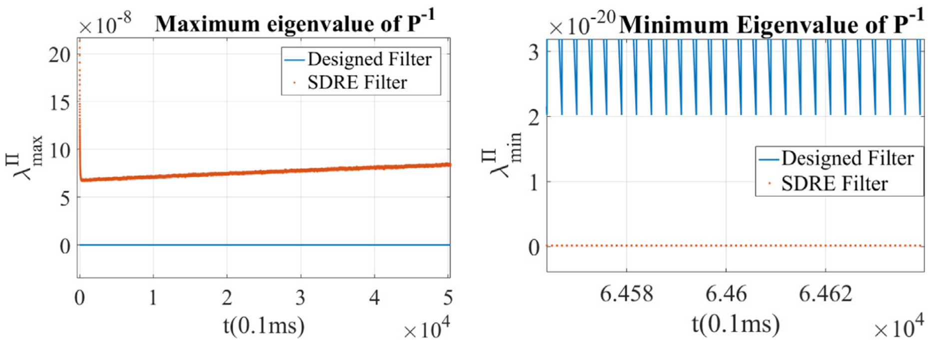


Fig. 5. Bound for estimation error in two cases, common filter and the designed filter.

gains. To discuss more accurately, Fig. 8b shows a magnified view of Fig. 8a which offers a good intuition on the designed filter performance; although its variance is higher than the other two methods through using switching, it is much lower in comparison with the variable structure filter.

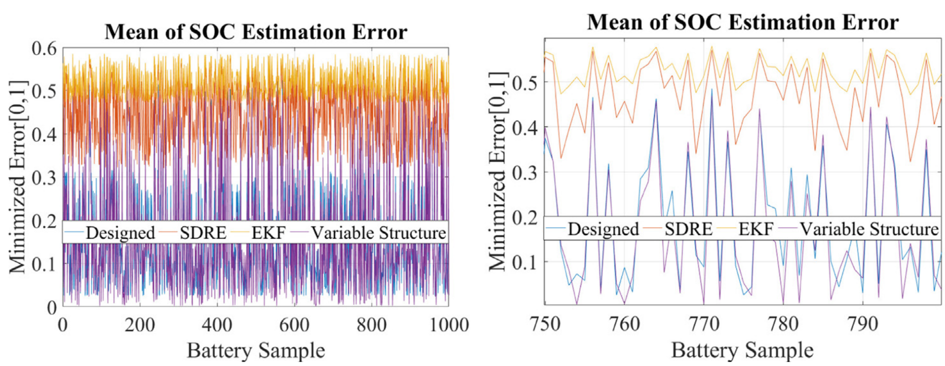
5.2. Experimental results

In this section, the proposed filter is implemented to estimate SOC of a Li-FePO4 battery cell. The battery model No. is 18650 and the nominal capacity and the rated voltage is 1400 mAh and 3.2 V respectively. To demonstrate performance of the proposed filter in SOC estimation, pulse discharge test is employed in which the current is switched. To clarify, Fig. 9 indicates the measured current. The model is tuned to work in Li-FePO4 battery range of voltage. To implement the filter, a PCB is designed and produced which is illustrated in Fig. 10. As it is seen, to increase the measurement accuracy, a 16-bit external ADC is utilized. Output voltage and discharge current of the cell is measured by this module. The sampling rate for this module is limited to 860 Hz that is adequate for the pulse discharge battery test as we have switched current with less frequency. Furthermore, to measure the current, a shunt resistor is utilized which is shown with a red bounding box in Fig. 10b; Note that the measurement sampling time is 0.06sec. Having these two variables, the state of charge can be estimated in real-time. The SPM model, used for SOC estimation, is highly sensitive to internal resistance of the battery. Thus, uncertainty in this parameter yields to



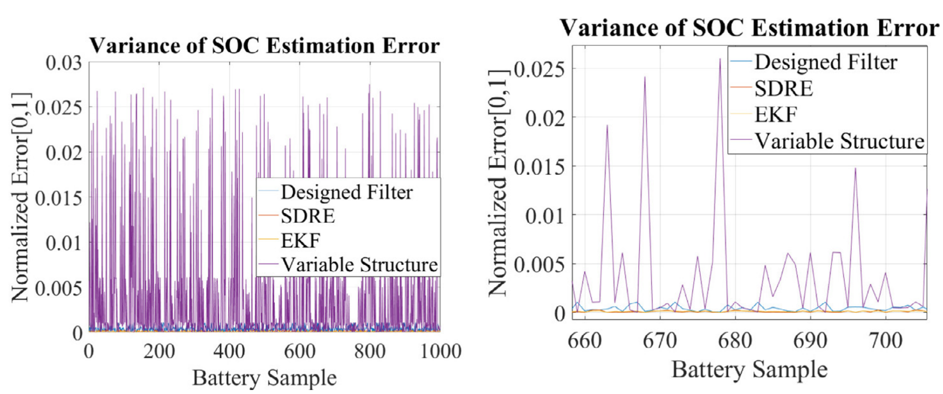
(a) λ^{Π}_{max} (b) λ^{Π}_{min}

Fig. 6. Minimum and maximum eigenvalues of matrix Π.



(a) (b) Magnified

Fig. 7. Mean of SOC estimation error as a result of Monte Carlo simulation.



(a) (b) Magnified

Fig. 8. Variance of SOC estimation error as a result of Monte Carlo simulation.

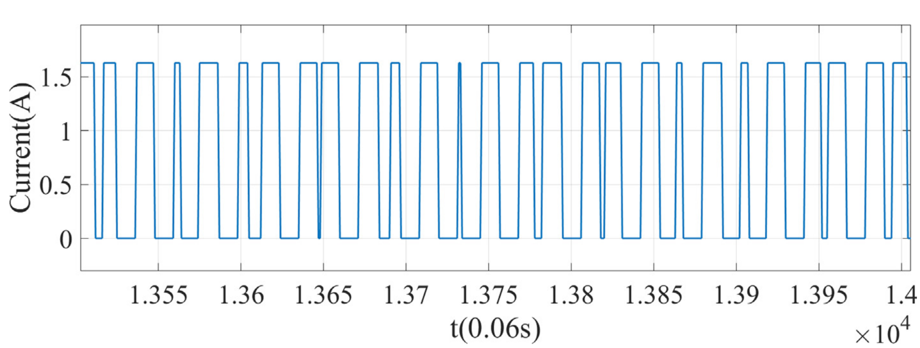
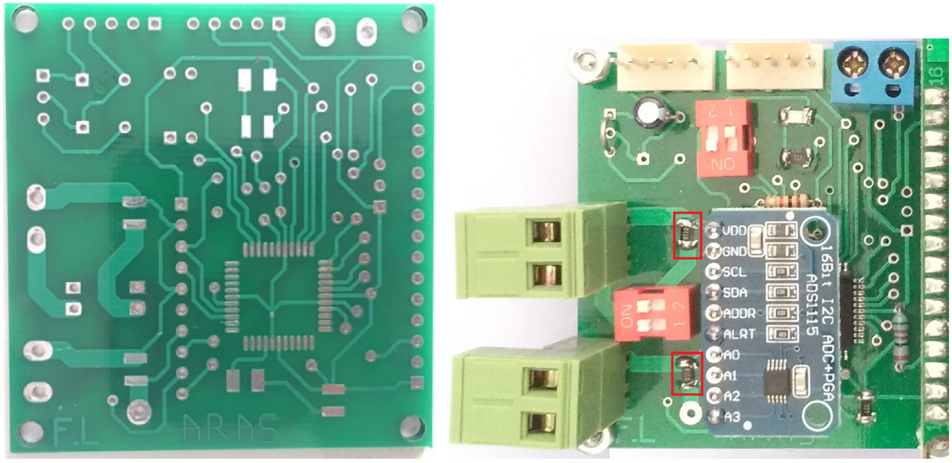
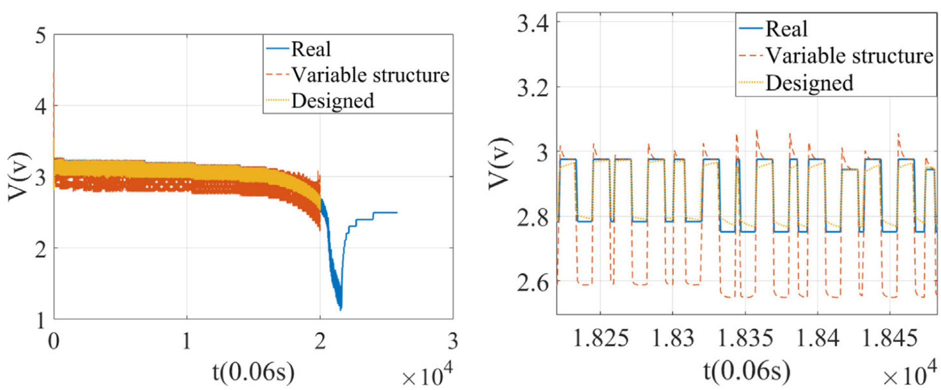


Fig. 9. The switched discharge current in the experiment.



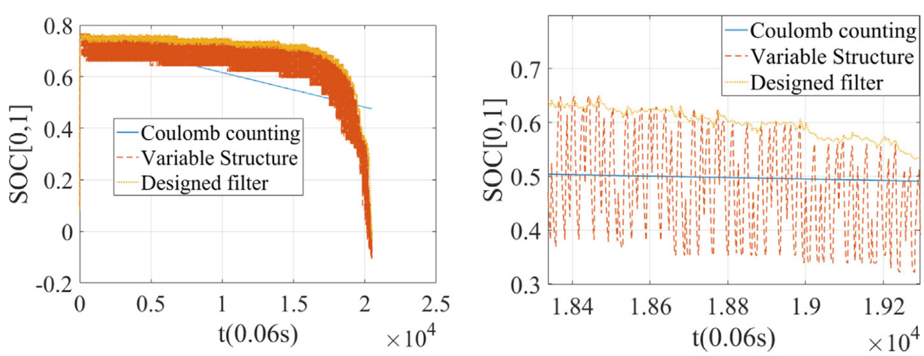
(a) (b)

Fig. 10. The designed and produced PCB to implement the proposed method.



(a) (b) Magnified

Fig. 11. Estimated battery terminal voltage for the proposed filter and variable structure method.



(a) (b) Magnified

Fig. 12. Estimated SOC for the three approaches.

large estimation errors. The following concept is used to measure the internal resistance approximately, in a way that uncertainty is minimized in this parameter. At the first sample time, when discharge current is zero, the SOC is estimated with a small error (without effect of internal resistance uncertainty(49)). In the next sample time, when the current is not zero, using the fact that SOC does not abruptly change, the internal resistance can be tuned to the approximately right value.

Remark 14. The most important limitation in filter implementation is due to the model (SPM), not the proposed filter; this model is not suitable for high currents.

The experimental result of measured and estimated battery terminal voltages, is shown in Fig. 11. The designed filter estimations closely agree with the actual voltage of the battery. Obviously, the filter cannot properly work for lower voltage levels (below minimum acceptable voltage of Li-FePO4), because of model limitations, and hence the estimation filter is executed in the first two-third time of the experiment. Note that, during battery discharge, the terminal voltage descends until the load removal, and is retrieved to a steady state after a while. Measuring the battery terminal voltage at this phase, yields estimated

SOC as approximately 35%. Hence, to optimize the battery life time, it is recommended to cut off the current whenever the estimated SOC is 35%; due to high rate of decrease in the terminal voltage level of battery.

To further investigate the effectiveness of the proposed filter, results for two common methods, variable structure and Coulomb approach are also reported. Fig. 12 shows the charge estimation results for the three approaches, Coulomb, switched-SDRE and variable structure filter. As it can be clearly seen in this figure, the estimations of the variable structure filter fluctuates more than the proposed filter. Furthermore, variable structure filter would have more fluctuations, if the current profile is pulsed. As a result, using the switched-SDRE filter is more reasonable to have less estimation variance. As it is indicated in Fig. 11, although the estimated voltage of the variable structure filter fluctuates more, the proposed filter estimation changes smoothly. Moreover, as shown in Fig. 12, Coulomb approach gives poor estimations of SOC; because this method requires precise knowledge of initial condition, and exact measure of charge/discharge current which is unavailable due to inaccuracy in current sensor. Furthermore, a lookup table is devised for cut-off SOC in this method; nevertheless, by environmental condition changes, cut-off SOC will be significantly altered. However, the proposed filter uses two quantities, voltage and the charge/discharge current in this regard. Thus, the SOC estimated by the proposed filter is much more accurate compared to that of the Coulomb method in variable environment using inaccurate current sensors.

Remark 15. As shown in Figs. 3, 4 and Figs. 7 and 8, the performance of the proposed filter and the variable structure filter is somewhat similar and superior to other filters. The distinguishing feature of both filters is their robustness to model uncertainties, which is more preferable in the proposed filter, as it shows less estimation fluctuations compared to the variable structure filter. The variable structure was used as a benchmark in experiments, since it is more robust compared to most of the common filters. As shown in Fig. 12, in experiments, the performance of the proposed filter is similar to the performance of the variable structure filter. However, the estimates obtained using the proposed filter have less variance compared to the estimates of variable structure filter, which evidences the superiority of the switched SDRE filter in terms of robustness. It is seen in Fig. 7 that the mean error of the estimation of the variable structure filter is slightly lower than the proposed filter, which is also evident in Fig. 12.

6. Conclusions

In this article, a switched SDRE filter was introduced for robust SOC estimation of Li-ion cells. Its stability was asserted applying direct Lyapunov method and dwell-time Theorems. Applicability of the proposed time-switching in robustness enhancement of conventional SDRE filter is fully addressed as stated in the Remarks 9 and 11. Effectiveness of this approach in estimation of SOC of Li-ion battery was verified through simulations using SPM model, and discussion over the comparison of four presented, common SDRE filter, EKF and variable structure filters on performance and robustness indicates that the presented filter is comparable to variable structure one. Moreover, experiments were conducted, which indicate this filter is more efficient and robust in comparison with common Coulomb approach, when using inaccurate current sensor, uncertain initial condition and in presence of environmental uncertainties.

Declaration of Competing Interest

The authors declare that there are no conflicts of interest.

Acknowledgment

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