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Coverage Control of Multi-Robot System for Dynamic Cleaning of Oil Spills

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***Abstract*— This paper addressed the dynamic cleaning of oil spills using a multi-robot system. The cleaning process is performed using a group of aerial agents adaptively covering an oil spill considering its advection and spreading on the sea surface. The concept of Voronoi Tessellation is employed to command the autonomous vehicles on how to adapt its configuration with the dynamic oil spill. Once the spill tracking error reduces to the desired value, all of the agents spray dispersant on the oil spill. This solution prevents wasting the excess amount of dispersant that is often being sprayed in conventional methods by aircraft. A distributed sliding mode control scheme is proposed to navigate the agents to the near-optimal Centroidal Voronoi Tessellation (CVT) by targeting the thick layers. Simulation results demonstrate that the proposed strategy can efficiently track the trajectory of the polluted area, and adaptively cover the time-varying shape of the oil spill.**

Keywords*— *Coverage control, Oil spill cleanup, Time varying environment, Multi-robot system

I. INTRODUCTION

Oil spills are a hazardous environmental problem that requires finding an effective cleanup solution. Among several types of cleanup methods, the dispersants known as the most common non-mechanical ones, have been demonstrated to be effective in this application [1,2]. Dispersants may be applied at high speeds by aerial vehicles such as aircraft and helicopters [3]; however, this spraying technique is not precise enough to focus only on the polluted area. Since the dispersants may be toxic, non-polluted areas not be touched by these chemicals as much as possible [4]. Therefore, proper treatment of oil spills with the dispersant necessitates a suitable understanding of the dynamic behavior of the polluted area. After employing an appropriate model to describe the oil spill behavior, a fast and accurate algorithm is required to track and cover the time-varying polluted.

One of the promising solutions for oil spill cleanup is the use of unmanned vehicles [5]. This idea dates back to the year 2010 when several novel methods such as Seaswarm [6] and Protei [7] were proposed. The implementation of Seaswarm requires special hardware that makes it difficult for implementation, and Protei is still undergoing complementary studies for practical implementation. Recently the concept of oil spill cleaning by unmanned vehicles is presented in [8], which provides complete coverage of the polluted area. The main drawback of the work reported in [8] is developing a navigation algorithm for only a single agent that makes the cleaning process time-consuming. Also, oil concentration and movement issues have not addressed. On the contrary, multi-agent cooperation reduces the time of oil cleanup mission by dividing the workspace area into several regions. In [9], a cooperative oil cleaning strategy for a team of surface autonomous vehicles has been proposed. Nevertheless, the essential issue of the dynamic behavior of the oil spill is not modeled. Furthermore, the motion strategy of the agents is in a simple back and forth motion that could result in further propagation of the contamination, and the agents are not able to optimally concentrate just on the polluted area that leads to excess energy consumption and longer mission time. Accordingly, efficient oil spill cleanup methods using a multi-agent system remain limited.

In order to overcome these restrictions, this paper presents a new strategy for oil spill cleaning by a group of aerial agents that can prevent dispersants wasting. Another benefit of the proposed strategy is that the aerial agents have no impact on the dynamic behavior of the oil spill; whereas the movement of the surface vehicles add undesired disturbance in the polluted area that makes the shape variation of oil spill more complex. The goal of the agents is tracking the dynamic changes in the shape and position of the oil spill. Cooperation between agents is achieved by the use of Voronoi partitioning. The employment of Voronoi partitioning for coverage problem was firstly introduced in [10], based on a gradient descent strategy. Later, a distributed solution in the presence of time varying density function was proposed in [11] simplifying the problem by assuming that initial position of the agents is at the Voronoi centroids. Unlike [11], reference [12] proposed a non-appointed initial condition method with single integrator dynamics that could be used in a limited range of linear systems. However, in recent years more realistic dynamic models have received increasing attention by researchers around the world to control agents by considering the nonlinearities. Sliding mode control owns outstanding properties such as robustness against parameter variations, model uncertainty and suitable rejection of environmental disturbances in nonlinear systems [13]. Sliding mode strategy is applied to reach the desired formation for a multi-agent system in finite time [14]. Inspired by [14], in this research, a fast sliding-mode control method is employed to achieve a finite-time coverage in which the agents navigated to the near-optimal Centroidal Voronoi Tessellation (CVT).

Dispersant work like detergents that are sprayed on oil spills to disperse them into the water column at low concentrations. The correct spraying equipment must be used to achieve the recommended treatment rate to prevent over-dosing or under-dosing [1]. Based on this requirement, this paper recommends using unmanned aerial vehicles (UAVs) in a group to provides a flexible coverage of the polluted environment in such a way that dispersants concentrate in the more polluted area. The proposed strategy in this work is capable of deploying the agents in the appropriate locations; i.e., more agents are present in the locations with higher contamination. The configuration of the agents is such that the encounter rate is improved and the dispersant hits the target spill at the desired dosage.

The main contribution of this paper is proposing a strategy in oil spill cleanup using an aerial multi-robot system that provides optimal coverage of the contaminated area preventing dispersants wasting, and developing a fast sliding mode control for a nonlinear multi-agent system to cover a time-varying environment. This control strategy is applicable to the general class of time-varying coverage applications. This article is organized as follows. Section 2 presents the oil spill modeling on the sea, while Section 3 describes the sliding mode control strategy for Voronoi-based coverage control in dynamic environments. In addition, this section provides theoretical proof of the proposed algorithm. A simulation scenario is given in section 4 to illustrate the effectiveness of the proposed control laws. The concluding remarks are given in Section 5.

II. Oil Spill Modeling

It should be possible to calculate the effect of spreading and advection as dominant oil fate processes on oil spills and thus establish how oil changes with time. The following section presents the formulation of oil spill modeling caused by the spreading and advection process in two dimensions in the dynamic sea environment.

A large number of oil spill models have been reported in the literature [15,16]. In this paper, the behavior of the oil spill modeled by the mechanical spreading and advection algorithms. This model consists of two main processes, including surface spreading due to the force of gravity and interfacial tension between oil and water, and furthermore, advection of the slick due to the water currents and wind drift. Now, let us declare the assumptions on oil modeling.

- Spillage stops before the start of the cleanup mission.
- The volume of oil is fixed and evaporation is neglected.
- Oil weathering processes are neglected.

The spill coverage is estimated by utilizing a relation developed by Berry [15], which showed the elliptical spreading of oil on the water's surface with the major radius being oriented in the direction of the wind. The area of the oil slick, A, may be given by:

$$A = (1/4)\pi QR \tag{1}$$

where Q and R are the lengths of the minor and major ellipse axes, respectively, determined by:

$$\begin{gathered} Q = 1.7(\Delta\rho V_o)^{1/3}\, t^{1/4} \\ R = Q + 0.03\,(U_{wind})^{4/3} t^{3/4} \\ \Delta\rho = \frac{\rho_w - \rho_{oil}}{\rho_w} \end{gathered} \tag{2}$$

in which, V_o is the volume of the spill, t is the time after the oil slick begins spreading, $\Delta\rho$ is the relative density difference between water and oil, U_{wind} is the wind speed and finally ρ_w and ρ_{oil} are the densities of water and oil respectively.

The spill generally moves at a speed that is 100% of the surface current and approximately 3.5-5% of the wind speed [2]. Therefore, the center of mass of the oil spill is advected according to the following equation:

$$U_a = U_{current} + 0.035 U_{wind} \tag{3}$$

where U_a is the resultant advective velocity of the oil spill; $U_{current}$ and U_{wind} are referred to as the surface currents and wind speeds respectively. The spill centers, which are determined by the resultant advective vectors, would now serve as the center of each ellipses [2], and the ellipses would orient themselves along the resultant vector U_a instead of wind in (2).

Moreover, the further away from the ellipse center, the thinner the oil slick becomes [2]. In this paper, the spreading phenomenon of the oil spill may be modeled using a Gaussian function with time-variant variance. The length of the major and minor diameters of the oil spill ellipse is equivalent to approximately 6 times the variance of the Gaussian function in the relevant directions.

Supposing a constant volume for the oil spill and assuming linearity, the distribution function of the pollution representing the thickness of oil on the water surface may be formulated by the following equation.

$$\phi(s,t) = \sum_{j=1}^{K} \alpha \left[exp\left(-\frac{(s_x(t)-x_{oil\_j})^2}{\sigma_x(t)}\right) + exp\left(-\frac{(s_y(t)-y_{oil\_j})^2}{\sigma_y(t)}\right)\right] \tag{4}$$

where K is the number of pollution sources, x_{oil_j}, y_{oil_j} are the position of j[th] spill center, σ_x and σ_y are calculated from (2).

In the next section, the formulation of coverage control will be extended for the dispersant cleanup method to enable a group of UAVs to target the oil spill with dispersant spraying.

III. Sliding Mode Coverage Control

This section presents the oil spill clean-up strategy which is based on Centroidal Voronoi Tessellation using a group of UAVs to cover the polluted area. The proposed algorithm provides complete coverage in such a way that in thicker oil areas more agents are present. Consequently, the dispersant method would be utilized in an optimal manner. Assume each agent is equipped with a downward spray with the spray angle α_i. Therefore, the limited actuation region of each agent is a circle denoted by $S_i = \{s \in R^2 : \|s - p_i\| \le r_i\}$ for agent in position $p_i \in R^2$. We also assume that a UAV can communicate and exchange information with other UAVs. The environmental conditions and the initial position of oil spill are predefined through remote sensing. It is also assumed that all UAVs are flying at the same altitude h, and we focus on planar control of agents in XY plane. Fig. 1 shows a conceptual prototype of UAV and its actuation region.

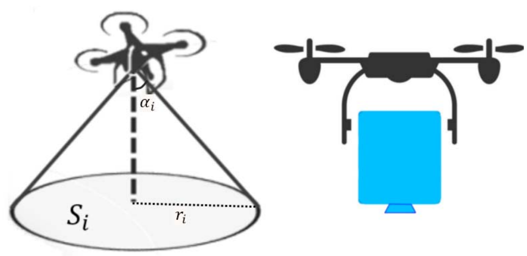


Fig. 1. UAV for Dispersant spraying

It should be noted that the minimum number of the required UAVs depends on three parameters including the spilled oil volume, dispersant type, and the loading capacity of each UAV. Let us consider a set of N agents where the dynamic of the ith agent described as

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = u_i + f_i(p_i, v_i) + g_i(t) \end{cases}$$
$$p_i = [x_i, y_i]^T \qquad (5)$$
$$i = 1, \dots, N$$

where $v_i \in \mathbb{R}^2$ is the velocity of the ith agent, u_i is the control input for the ith agent, $g_i(\mathrm{t})$ is bounded external disturbance, and f_i is the nonlinear dynamic of the i^{th} agent.

Each agent partitions the convex workspace $G \in R^2$(area to be covered) according to Voronoi tessellation technique presented in [10], such that $\bigcup_{i=1}^{n} V_i = G$.The position of agents $P = \{p_1, p_2, \dots, p_n\}$generate a Voronoi diagram that is defined with $V = \{V_1, V_2, \dots, V_n\}$. It is supposed that each agent covers its domain V_i. As we move away from point $p_i$inside the mission space, the actuation performance of the i^{th} agent is reduced with distance because of the limited actuation range of spraying. Hence, the performance function of i^{th} agent is defined by function $k: \mathcal{R}^+ \to \mathcal{R}^+$. The distribution density function of an oil spill is modeled by a time-varying function defined as $\phi(s,t)$. Therefore, the framework of locational optimization is utilized and the coverage cost function is defined as

$$H(P,Q) = \sum_{i=1}^{N} H(p_i, V_i)$$
$$= \sum_{i=1}^{N} \int_{V_i} k(\|s - p_i\|)\, \phi(s,t)\, ds \qquad (6)$$

The coverage cost function must be minimized with regard to the location of agents and Voronoi partitioning. By differentiating the coverage cost function with respect to the its pose p_i, The local minimum points for this function are the centroids of Voronoi cells determined as

$$C_{V_i} = \frac{\int_{V_i} s\, \phi(s,t) ds}{\int_{V_i} \phi(s,t) ds} \qquad (7)$$

We refer to the agents' configuration as a Centroidal Voronoi Tessellation (CVT) if they reach the centroid of their Voronoi partitions [10]. Whenever ϕ is time-variant, the CVT will be time-variant as well. Hence, not only the CVT should be reached but also, the agents should track the time-varying CVT. Consequently, optimal deployment can be transformed into a desired point tracking problem. Note that since each robot moves toward its own Voronoi center, no one can leave its Voronoi cell and there is no possibility of collision among them. The tracking control strategy starts with defining the tracking error which represents the agent distance from the centroid of its Voronoi cell as

$$e_i = p_i - C_{V_i} \qquad (8)$$

The objective of the designed distributed sliding mode control is to converge the tracking error toward zero for individual agents in finite time. The tracking error dynamics are obtained by taking the second time derivative of (8) as

$$\begin{cases} \dot{z}_{1i} = z_{2i} \\ \dot{z}_{2i} = u_i + f_i(p_i, v_i) + g_i(t) - \ddot{C}_{V_i} \end{cases} \qquad (9)$$

where $e_i = z_{1i}$ and $\dot{e}_i = z_{2i}$. Now consider the sliding surface as a vector function $\sigma: R^N \times R^N \to R^N$given by

$$\sigma(z_1, \dot{z}_1) = \dot{z}_1 + \beta\, S(z_1)|z_1|^{1/2}$$
$$z_1 = [z_{11}, \dots, z_{1N}]^T$$
$$|z_1|^{1/2} = \left[|z_{11}|^{\frac{1}{2}}, \dots, |z_{1N}|^{\frac{1}{2}}\right]^T \qquad (10)$$
$$S(z_1) = diag[sign(z_{11}), \dots, sign(z_{1N})]$$

where $\sigma = [\sigma_1, \dots, \sigma_N]^T$,$\sigma_i \in R$, $\beta = diag[\beta_1, \dots, \beta_N]$, $\beta_i > 0$, $z_{1i} \in R$, $z_1 \in R^N$.

As in conventional sliding mode control, the control law u_i is calculated by setting $\dot{\sigma} = 0$ for the nominal system and adding a signum function to guarantee finite-time convergence to the sliding surface. Consequently, the control signal will be

$$u_i = \ddot{C}_{V_i} - f_i(p_i, v_i) - \tfrac{1}{2}\beta_i \dot{z}_{1i}|z_{1i}|^{\frac{-1}{2}} - M_i sign(\sigma_i) \qquad (11)$$

where $\sup(\|f_i(p_i, v_i) + g_i(t)\| + \lambda_i) \le M_i$. $H_i \subset R \times R$ is the set where $\left\|\dot{z}_{1i}|z_{1i}|^{\frac{-1}{2}}\right\|$ is bounded. Next, H_i define as

$$H_i \triangleq \left\{(z_{1i}, \dot{z}_{1i}) \in R^2 \times R^2 \colon \left\|\dot{z}_{1i}|z_{1i}|^{\frac{-1}{2}}\right\|_\infty \le \lambda_i\right\} \qquad (12)$$

Based on the analysis in [14], the initial conditions must be inside H_i such that $\lambda_i = \|\beta\|_\infty + \delta_i$ and $\delta_i > 0$. The result presented in [14] discussed the sufficient conditions under which H_i is positively invariant and the controller in this region of the state space is bounded [17]. Therefore, for initial conditions in this region, the controller guarantees convergence of closed-loop system trajectories to the desired value in finite time. The term $\ddot{C}_{V_i}$ represents the time-varying behavior of the environment. The stability of closed-loop system will be investigated in the following theorem.

Theorem 1. Considering the multi-robot system described by (5) with the sliding surfaces given in (10), if the control inputs adopted as given in (11), then the tracking errors will converge to zero in a finite time, and consequently, the area is covered by the agents in finite-time.

Proof. Consider a Lyapunov function candidate given by

$$\mathrm{V}(\sigma) = \tfrac{1}{2}\sigma^T\sigma \qquad (13)$$

where σ is defined in (10). Note that if $(z_{1i}, \dot{z}_{1i}) \in \{(z_{1i}, \dot{z}_{1i}) |\ \sigma = 0\}$, then $\mathrm{V}(\sigma) = 0$, otherwise $\mathrm{V}(\sigma) > 0$. Differentiate Lyapunov function along the trajectory given in (9).

$$\dot{V}_i = \sigma_i\dot{\sigma}_i = \sigma_i\left(u_i + f_i(p_i, v_i) + g_i(t) - \ddot{C}_{V_i} + \tfrac{1}{2}\beta_i\dot{z}_{1i}|z_{1i}|^{\frac{-1}{2}}\right)$$
$$\dot{V}_i = \sigma_i\big(g_i(t) - M_i sign(\sigma_i)\big) \le -M_i|\sigma_i| \qquad (14)$$
$$\dot{\mathrm{V}}(\sigma) \le -\sqrt{2}\min_{i=1,\dots,N}(M_i)\,(\mathrm{V}(\sigma))^{\frac{1}{2}}$$

Thus, based on Theorem 2.2 of [18], the trajectories of (9) converge to the sliding surface $\sigma = 0$ in a finite time and remain on it. Furthermore, while on the sliding surface, the closed-loop dynamics are given by

$$\dot{z}_{1i} = -\beta\, S(z_{1i})|z_{1i}|^{1/2} \qquad (15)$$

Now, consider the Lyapunov function candidate $\bar{V}(z_1) = \|z_1\|_1$, $z_1 = [z_{11}, \dots, z_{1N}]^T$, and differentiate it along the trajectories (15) as

$$\dot{\bar{V}} = \sum_{i=1}^{N} sign(z_{1i})\, \dot{z}_{1i}$$
$$= -\sum_{i=1}^{N} \beta_i|z_{1i}|^{\frac{1}{2}} \le -\|\beta^{-1}\|_\infty^{-1}(\bar{V}(z))^{1/2} \qquad (16)$$

Thus, using Theorem 2.2 in [18] implies finite-time convergence of the error to the origin. That means the proposed sliding mode controller guarantees error convergence to zero in finite time, which represents the finite-time area coverage of the multi-agent system.

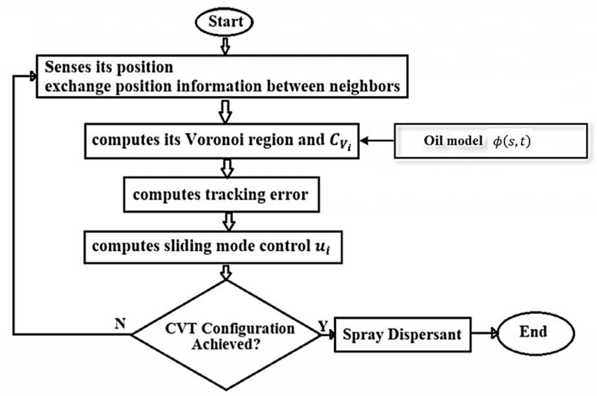


Fig. 2. Sliding mode coverage control algorithm for oil spill cleanup

The proposed sliding mode coverage control algorithm summarized in Fig. 2. Based on this algorithm, the closed-loop system converges to a Centroidal Voronoi configuration and after then the dispersants sprayed on the oil spill.

Remark. The controller design process will use the variable $\ddot{C}_{V_i}$. To avoid numerical differentiation of C_{V_i}, the following second-order filter is introduced:

$$\ddot{C}^{\alpha}_{V_i} + 2\xi\theta\dot{C}^{\alpha}_{V_i} + \theta^2\left(C^{\alpha}_{V_i} - C_{V_i}\right) = 0 \tag{17}$$

where $\ddot{C}^{\alpha}_{V_i}$, $\dot{C}^{\alpha}_{V_i}$ and $C^{\alpha}_{V_i}$ are the estimate values corresponding to $\ddot{C}_{V_i}$, $\dot{C}_{V_i}$ and C_{V_i} respectively. Therefore, in the formulas, $\ddot{C}_{V_i}$will be replaced by $\ddot{C}^{\alpha}_{V_i}$.

Remark. To remedy the effect of the chattering problem due to the signum function, this function may be replaced with a saturation function as

$$Sat(\sigma_i,\varepsilon) = \begin{cases} \sigma_i/\varepsilon & if \ \ |\sigma_i| < \varepsilon \\ \sigma_i/|\sigma_i| & if \ \ |\sigma_i| \geq \varepsilon \end{cases} \tag{18}$$

where boundary layer thickness is 2ε, in which ε is a positive constant. Consequently, the ultimate boundedness of the tracking errors will be obtained.

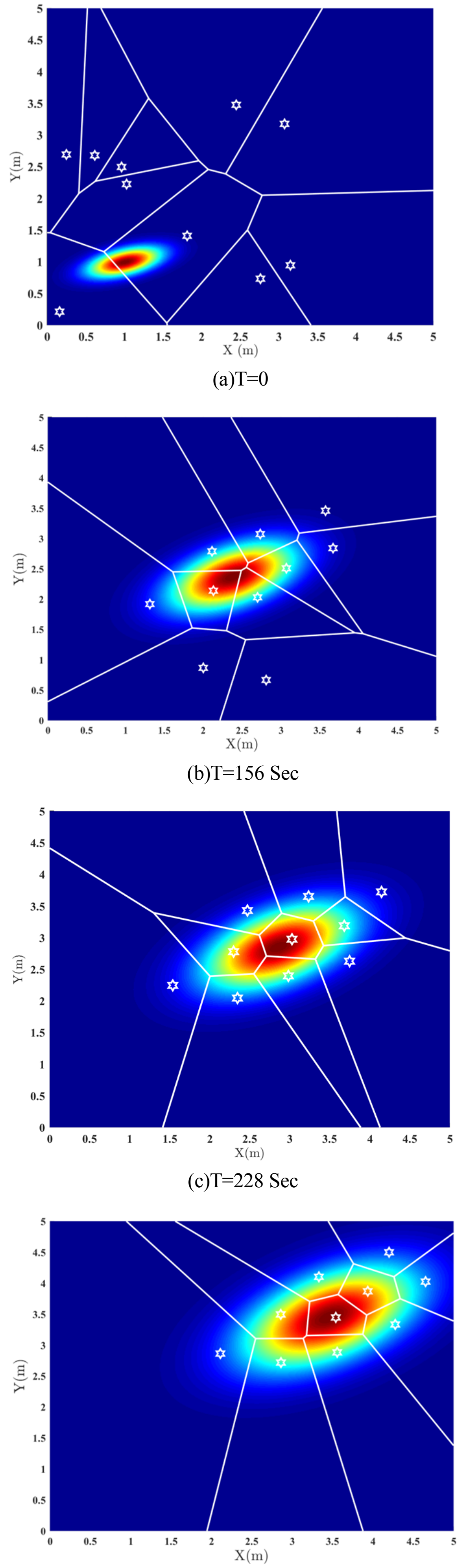
IV. Simulation and Result

This section presents numerical simulations for 10 agents represented by double integrators to demonstrate the effectiveness of the proposed control algorithm. The agents' dynamics are given by,

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = u_i + \alpha \, sin(\omega t) \end{cases} \tag{19}$$

where $\alpha = 0.5$ and $\omega = 2$. Note that the behavior of UAVs can be represented by a simple double integrator model with bounded velocity and acceleration [19,20]. The numerical parameters of the oil spill model are given in Table I. The control coefficients in are chosen as $= diag[2.\dots.2]$, $\xi = 1$, $\theta = 5$, $M_i = 5$ for $i = 1.\dots.10$. The surface current of water in this area is toward the north while the wind blows toward the north-west.

Unlike [11] in which the initial positions of the agents must be adopted based on the CVT, in the given simulations in Fig. 3 this restriction is removed, and they are set randomly in two dimensions at a rectangular bounding box nearby the oil spill.



(a)T=0

(b)T=156 Sec

(c)T=228 Sec

(d)T=312 Sec

Fig. 3. UAVs configuration versus time

TABLE I. SIMULATION PARAMETERS

Parameter	Value	Unit
ρ_{oil}	0.85	g/cm3
ρ_w	1.03	g/cm3
V_o	100	Bbl
N	10	-
U_{wind}	6.43	m/min
$U_{current}$	0.38	m/min
α_i	30	Degree
h	1	m
ε	0.1	-

Fig. 3 illustrates the top view of the oil spill tracking and coverage by the multi-agent system. In Fig. 3 (a) the agents are located in their randomly initial positions indicated by white stars and the oil spill could be observed in the left bottom of the mission space. Fig. 3 (b) shows the situation after 156 seconds, where the moving oil spill has been enlarged and the agents have become closer to it. Figures (c) and (d) indicate the situation at times 228 and 312 seconds respectively, when on the last occasions the coverage criterion is successfully met then the dispersant spraying should be started and the mission ends. It can be seen from the figure that the oil spill has become greater due to the spreading effect and movement caused by the advection. The dynamic nature of the proposed technique adjusts the agents' positions concerning the variations in the shape and trajectory of the polluted area and maintains the optimal configuration for the multi-robot system by deploying more agents in the points with more contamination.

Fig. 4 shows the centroid of the Voronoi cell for each agent versus time. The variations in the centroid of the Voronoi cells in this figure are due to the behavior of the oil spill on the sea surface. The tracking errors of the agents in (8) are illustrated in Fig. 5. It is seen from this figure that the tracking errors approach to zero in a finite time.

Fig. 6 shows the coverage cost of the proposed control method versus time. As seen from the figure, the coverage cost reduced over time until it reaches the minimum level, and never converge to zero due to the dynamic time-varying behavior of the oil spill and thus, the agents are continuously moving to cover new points.

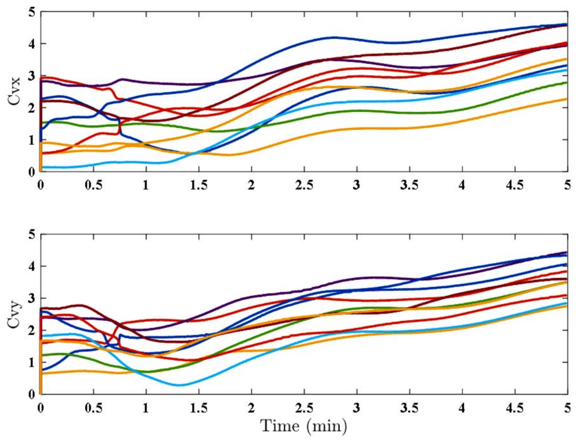


Fig. 4. Centroid of Voronoi cells versus time1

Fig. 7 shows the actuation region of each agent indicated by a circle. As it was expected, the actuation regions have more overlaps in the more polluted area, which means more dispersant will be provided for more contaminated parts. In this case, the agents are located proportionally to the pollution thickness. That means in more polluted areas, more doses of dispersant will be sprayed.

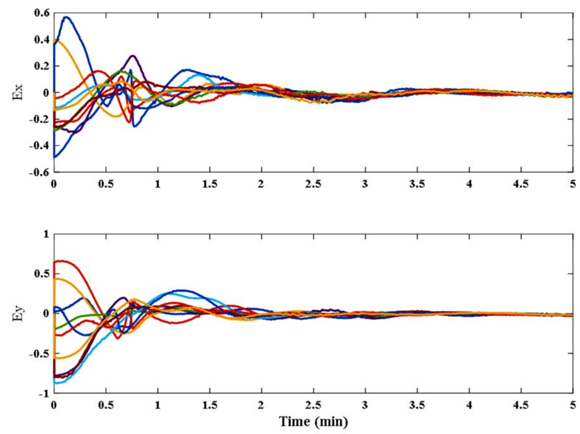


Fig. 5. Tracking error of the UAVs over time

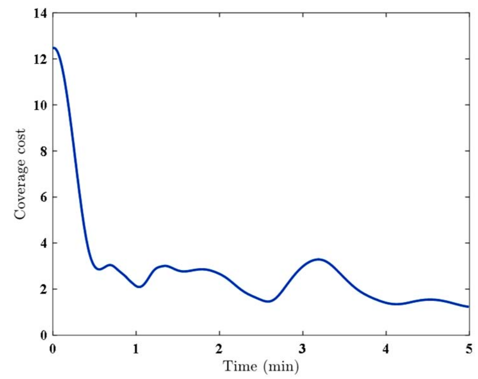


Fig. 6. Coverage cost versus time

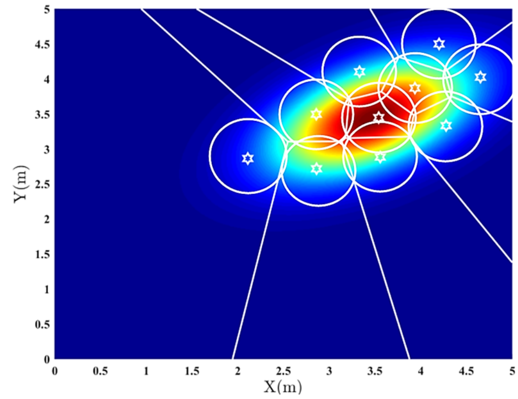


Fig. 7. Actuation regions of the multi-robot

V. CONCOLUSIONS

In this paper, an area coverage algorithm in oil spill cleanup mission has been proposed with the sliding mode control method, considering dynamic behavior of oil spill on the sea environment. The multi-robot system converges to the optimal configuration based on the idea of Voronoi diagram such that the dispersants hit the target oil at the desired dosage. The UAVs deployment has been performed according to the thickness of the polluted area that improves the efficiency of dispersant method. Simulation results demonstrated the effectiveness of the proposed control algorithm in finite-time area coverage in a dynamic environment. In the future, more work should be considered to research hardware implementation and take into account the robustness of the proposed controller against external disturbances.

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