بنام آنکه جان را فکرت آموخت

کنترل غیرخطی

تمرین سری اول

دانشگاه صنعتی خواجه نصیرالدین طوسی ۱۳۰۷

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دانشکده مهندسی برق

گروه کنترل و سیستم

نیم‌سال اول ۱۳۹۹–۱۴۰۰

موعد تحویل: ۹۹/۰۷/۲۰

Exercise 1: The magnetic levitation experiment is a mechatronic system that consists of an electromagnet, a ball and a post encased in a rectangular enclosure as shown in Figure 1.

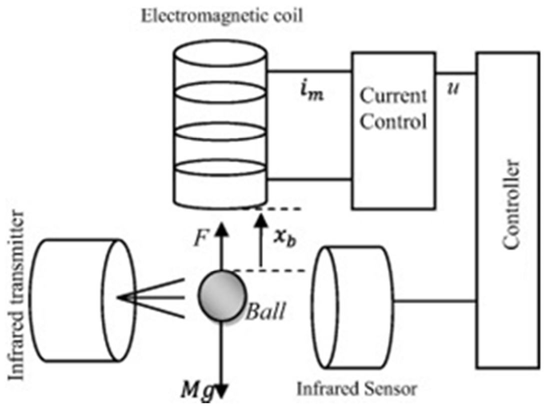
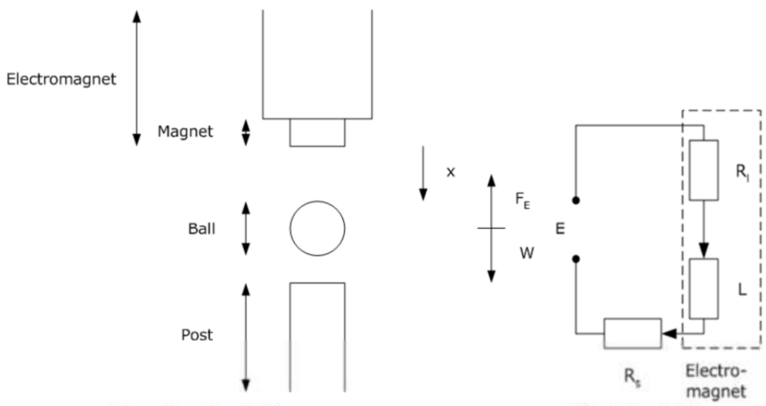


Figure1: magnetic levitation experiment setup

1- By using Newton's second law and Kirchhoff's current loop, derive the state space model $\dot{x} = f(x,u)$ of this mechatronic system. Use $x = [x \quad \dot{x} \quad i]^T = [x_1 \quad x_2 \quad x_3]^T$ and $u = E$.

(Hint: you can use $\Sigma F = m\ddot{x} = W - F_E$ and electromagnetic force as $F_E = K\frac{i^2}{(x+d)^2}$,and below diagrams for more information).



Mechanical Diagram **Electrical Diagram**

2- Derive the equilibrium points (x^*, u^*)

3- Using the following parameters, find the stability type of the equilibrium points.

$$R_i = 10\ ohm\ , R_s = 1\ ohm\ , L = 0.4125, g = 9.81\ m/\ s^2\ , x_1^* = 7.5mm$$

Exercise 2: Consider the system described by following differential equations

$$\dot{x}_1 = x_1^2 x_2 + x_1 x_2^2$$

$$\dot{x}_2 = (-x_1 + x_2)(x_2^2 - 1)$$

1- Find equilibrium points of this nonlinear system and linearize the system for all the equilibrium points and specify the type of each equilibrium point.

2- Sketch the phase portrait, using MATLAB toolbox functions, and verify the behavior of the equilibrium points you obtained in the previous section.

Exercise 3: The dynamics of a nonlinear system are described by

$$\dot{x}_1 = \mu - x_1(x_1^2 - 1)$$

$$\dot{x}_2 = -x_2$$

where μ is the perturbation parameter.

1- Find all the equilibrium points of the system.

2- Linearize the system about each equilibrium points and discuss the qualitative behavior of the system as the parameter μ varies.

Exercise 4: Consider the following nonlinear system

$$\begin{cases} \dot{x} = -x - \dfrac{y}{ln\sqrt{x^2 + y^2}} \\ \dot{y} = -y + \dfrac{x}{ln\sqrt{x^2 + y^2}} \end{cases}$$

1- Find the equilibrium points of the system. Linearize the system around the obtained equilibrium points and specify the type of their stability.

2- Sketch the phase portrait using Matlab toolbox functions, and verify the stability of the equilibrium point, and existence and type of any limit cycle, and its .

3- Simulate the system and plot x and y with initial value $[0.5,0.5]$ in time and phase portrait, and verify convergence to the equilibrium point. Simulate the system with the initial value of $[1.5,1.5]$. plot the response in time and phase portrait. Verify the behavior in time domain, and determine the frequency of the oscillations as times goes to infinity?

xplain why the response frequency is high frequency with the initial condition $[1.5,1.5]$. (you can use polar coordinates.)

Exercise 5: Consider the following system:

$$\dot{x_1} = k_1 x_2 (1 - \frac{x_1}{1 + {x_2}^2})$$

$$\dot{x_2} = k_2 - x_2 - \frac{4 x_1 x_2}{1 + {x_2}^2}$$

Where $k_2 > 0, k_1 > 0$ are positive constants.

1- Find the equilibria of this system.

2- Derive a condition under which the system is guaranteed to have a periodic orbit inside the region

$$K = \{(x_1, x_2) | x_1 \geq 0, x_2 \geq 0, x_1 \leq 1 + {k_2}^2, x_2 \leq k_2\}$$

(i.e. show invariance of K and derive the condition under which the equilibrium point is unstable)

3- Using this, describe the type of bifurcation that occurs as the parameter k_1 varies.

Extra Exercise for interested students (Bonus mark 15%):

By using nonlinear differential equations, we can not only describe and analyze biological, economical and other types of systems, as well. As an example, a simplified representation of populations of foxes and rabbits is given by the following equations:

$\dot{x} = x(\alpha - \beta y)$

$\dot{y} = -y(\gamma - \delta x)$

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Where:

x: Number of rabbit

αx: Natural reproduction of rabbits

βxy: Rate of predation

y: Rate of predation

γy: Natural death of foxes

δxy: Growth rate for foxes

$\alpha, \beta, \gamma, \delta$: Parameters representing the interaction of the two species

We know that $x \geq 0; y \geq 0$, since the number of animals cannot be negative. Note that the rate of predation is quite similar to the growth rate for foxes, but they contain different parameters (as the fox population growth is not ***necessarily*** equal to the rate at which it consumes the rabbits).

1- If initially $x > 0; y = 0$, what would happen to the rabbit population?

2- If initially $x = 0; y > 0$, what would happen to the fox population?

3- Find the equilibrium points of the system, and determine the type of each equilibrium point when all parameters are positive.

4- Construct the phase portrait for $x \geq 0; y \geq 0$ and discuss the qualitative behavior of the system. Choose the values $\alpha = 63; \beta = 5; \gamma = 457; \delta = 6$.

5- If initially $x > 0$ and $y > 0$, is it possible to arrive at $x < 0$ or $y < 0$? Explain your answer. (Hint: See definiton in Slotine at p. 68)

6- Using Bendixson's criterion (Lemma 2.2), specify a *simply connected* region in which *no* periodic orbits are lying entirely. You may describe the region using inequalities. (Hint: See definiton in Khalil at p. 67, Slotine at p. 37)