دانشکده مهندسی برق
گروه کنترل و سیستم
نیم‌سال اول ۱۳۹۹–۱۴۰۰
موعد تحویل: ۹۹/۰۸/۰۴

بنام آنکه جان را فکرت آموخت

کنترل غیرخطی

تمرین سری دوم

۱۳۰۷
دانشگاه صنعتی خواجه نصیرالدین طوسی
استاد: حمید رضا تقی راد

Exercise 1: Analyze the stability of the origin as the equilibrium point of the following system.

$$\dot{x}_1 = -x_1 + x_2^6$$

$$\dot{x}_2 = x_1^6 + x_2^3$$

Exercise 2: Using the Variable Gradient Method, find a Lyapunov function to show the origin is globally asymptotically stable equilibrium point for the following system.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -(x_1 + x_2) - h(x_1 + x_2)$$

Where $h(0) = 0$ and $h(z) > 0 \ , \forall \ z \neq 0$.

Exercise 3: Consider the following nonlinear system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{(\gamma + \beta {x_2}^2)\sin(x_1)}{\alpha + \beta\cos(x_1)} - u_d$$

$$\dot{x}_3 = \frac{2(\gamma + \beta {x_2}^2)\sin(x_1)}{\alpha + \beta\cos(x_1)} + u_d$$

where α , β and γ are constant and positive and $|x_1| < \frac{\pi}{2}$

The control input is designed as follows:

$$u_d = \frac{2x_2}{(\alpha + \beta \, cos(x_1))^2} + \rho(2x_2 + x_3)$$

where ρ is a positive constant. Use the following Lyapunov function to analyze the stability of the equilibrium point.

$$V = \frac{\gamma + \beta {x_2}^2}{\beta(\alpha + \beta \, cos(x_1))^2} - \frac{\gamma}{\beta(\alpha + \beta)^2} + 0.5\rho(2x_2 + x_3)^2$$

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Exercise 4: Consider the following nonlinear system

$\dot{x}_1 = x_2$

$\dot{x}_2 = -x_2 - sin(x_1) - 2sat(x_1 + x_2)$

1. Show that the origin is the unique equilibrium point of the system.

2. Use linearization method to analyze the stability of the origin.

3. Take $= x_1 + x_2$, and show for $|\sigma| \geq 1$ we have : $\sigma\dot{\sigma} \leq -|\sigma|$

4. Consider the function $V = {x_1}^2 + 0.5{x_2}^2 + 1 - \cos(x_1)$ and the set M_c defined as

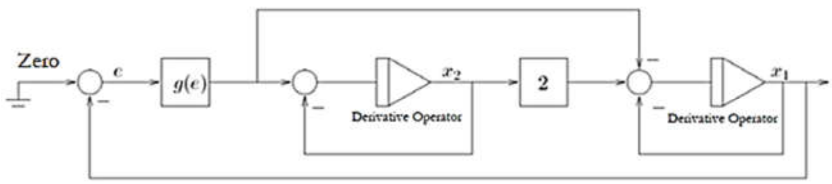
$$M_c = \{x \,\epsilon\, R^2 | V(x) \leq c\} \cap \{x \,\epsilon\, R^2 |\; |\sigma| \leq 1\}.$$

Show that M_c is a positive invariant set and every path inside M_c converges to the origin as $t \to \infty$.

5. Show that origin is globally asymptotically stable.

Exercise 5:

1. Consider the system in below figure, where the nonlinear function is given by $g(e) = e(t)^3$.



a. Find the state space model.

b. Show that the origin is asymptotically stable using the Lyapunov function:

$$V(x) = x^T P x$$