بنام آنکه جان را فکرت آموخت

کنترل غیرخطی

آزمون میانی

۱۳۰۷
دانشگاه صنعتی خواجه نصیرالدین طوسی

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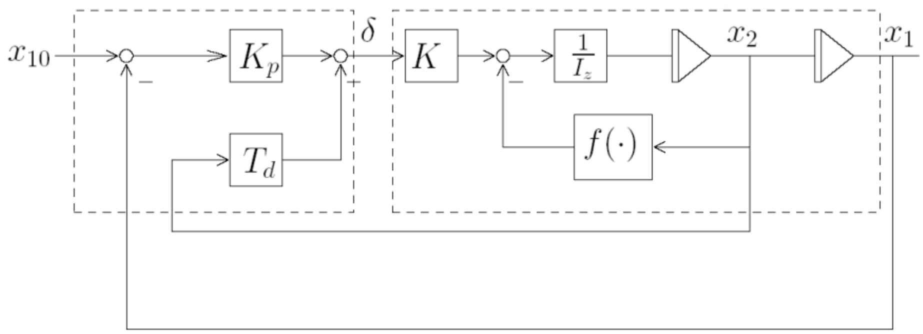
گروه کنترل و سیستم

نیم‌سال اول ۱۴۰۰–۱۳۹۹

زمان آزمون: ۲ ساعت + ۱۵ دقیقه برای بارگذاری

PROBLEM 1: (30%)

For the following feedback control system,



In which, $K = I_z = 1$, $f(x_2) = -x_2 + x_2|x_2|$, and the system parameters $K_p > 0$ and $T_d > 0$ are positive, the system dynamics may be written as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = K_p\,(x_{10} - x_1) - (T_d - 1)x_2 - x_2\,|x_2|$$

1. Find the equilibrium point(s) of the system.
2. Derive the type of equilibrium point(s) expressed in terms of K_p and T_d, and where it might become unstable.
3. Select $K_p = T_d = x_{10} = 1$, specify find the type of equilibrium point, and verify your solution by drawing the system phase portrait in Matlab.

PROBLEM 2: (20%)

Consider the following nonlinear system for some constatnt parameters a, b:

$$\dot{x} = a\,xy^2 + y\cos(x)$$

$$\dot{y} = b\,x^2 y + \frac{1}{2}y^2\sin(x)$$

1. Determine for what values of a, b, there exists no limit cycle in the system trajectory.
2. Check the conservatism of your result for some parameters a, b that do not satisfy the above conditions by drawing the phase portrait of the system in Matlab.

PROBLEM 3: (25%)

Consider the following nonlinear system

$$\dot{x}_1 = -\frac{x_2^3}{1 + x_2^8}$$

$$\dot{x}_2 = x_1^3 - x_2$$

By using a Lyapunov function candidate as : $V(x) = x_1^4 + \psi(x_2)$, where ψ shall be suitably selected, analyze the stability of the origin.

PROBLEM 4: (25%)

Consider the following non-autonomous systems

$$\dot{e} = Ae + (\hat{K} - K)\phi(t)$$

$$\dot{\hat{K}} = -\phi(t)e^T P$$

where A is Hurwitz, and $e \in \mathbb{R}^2, \hat{K} \in \mathbb{R}^2$ are the state variables, $K \in \mathbb{R}^2$ is a constant vector of unknown gains, and $\phi(t) \in \mathbb{R}^2$ is assumed to be bounded. In this system $P \in \mathbb{R}^{2\times2}$ is the symmetric positive definite solution of the Lyapunov equation $PA + A^T P = -Q$, in which Q is a symmetric positive definite matrix. Check uniform boundedness of the variables and analyze the stability of the origin.