کنترل غیرخطی

تمرین سری پنجم

۱۳۰۷
دانشگاه صنعتی خواجه نصیرالدین طوسی
استاد: حمید رضا تقی راد

Exercise 1:

Consider the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} {x_2}^2 + u \\ {x_3}^2 + u \\ p(x_1) + u \end{bmatrix}$$

1. Find all functions $p(x_1)$ for which the system is full state-feedback linearizable near the origin.
2. For each function obtained from the previous section, find the control rule u in such a way that the origin of the above system is locally asymptotic stable.

Exercise 2:

An important class of nonlinear systems can be written in the form

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\vdots$$
$$\dot{x}_n = f(x) + g(x)u$$

Assume that the full state x is available for measurement.

1. Find a feedback:

$$u = h(x, v)$$

that renders the closed loop system from the new input v to the state linear. What conditions do you have to impose on $f(x)$ and $g(x)$ in order to make the procedure well-posed?

2. Apply this procedure to design a feedback for the inverted pendulum

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \mathrm{a}\sin(\mathrm{x}_1) + \mathrm{b}\cos(\mathrm{x}_2)\mathrm{u}$$

that makes the closed loop system behave as a linear system with a double pole in $s = -1$. Is the control well defined for all x? Can you explain this intuitively?

3. One drawback with the above procedure is that it is very sensitive to modelling errors. Show that this is the case by designing a linearizing feedback for the system

$$\dot{x}_1 = x^2 + u$$

that makes the closed loop system linear with a pole in -1. Apply the suggested control to the system

$$\dot{x}_1 = (1+\epsilon)x^2 + u$$

and show that some solutions are unbounded irrespective of $\epsilon \neq 0$.

Exercise 3:

The technique of an input-output feedback linearization is used for many control applications, which transforms a complicated and coupled nonlinear system into an equivalent linear system that could be controlled using linear control system tools, such as pole placement technique. Thus, to find such control law a MATLAB script is developed to facilitate the tedious computations and avoiding errors due to a complexity of the system.

1. Let's consider the following nonlinear mathematical system representation:

$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i$ (1)

$$y_j = h_j(x)$$

The flow chart of Figure 1 gives a detailed explanation of how the program works; therefore, the following steps are used to obtain the nonlinear control law using the developed program.

a) Introduce the nonlinear functions $f(x), g(x)$ and $h(x)$ as a symbolic functions to the program

b) Find the lie derivative along the vector fields $f(x)$ and $g(x)$

c) If the lie derivative along the vector fields $g(x) = 0$ and the number of differentiation is less than n (the system state number), then set $f(x) = Lf\ h(x)$ and repeat step 2

d) If the lie derivative along the vector fields $g(x) = 0$ and the number of differentiation is less than n (the system state number), then set $f(x) = Lf\ h(x)$ and repeat step 2

e) If the lie derivative along the vector fields $Lgh(x) = 0$ and the number of differentiation is equal or greater than n (the system state number), then the system does not admit an input output feedback linearization

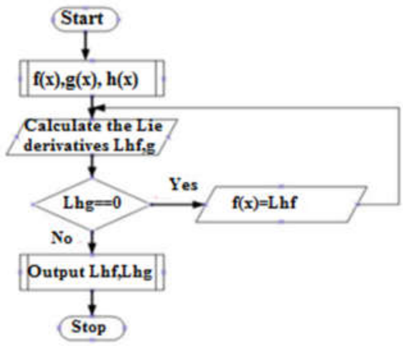

Figure 1. *The flow chart of the proposed algorithm*

Use your code to solve this simple SISO systems and verify your answer by hand calculation.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\sin(x_1) + u$$

2. Extent this code to extract the linearization control law. Use your code for the following nonlinear dynamical system and verify your answer by hand calculation.

$\dot{x} = f(x) + g(x)u = \begin{bmatrix} x_1 + x_1x_2 \\ -\sin(x_1) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, The output vector of the system is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

3. Let us determine the associated internal dynamics of the aircraft:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -4x_2 - 4x_1 + 3E$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = 6x_1 - E$$

$$y = x_3$$

Where E is the input vector. By applying the MATLAB-based program, derive the feedback linearization controller law.

دانشکده مهندسی برق گروه کنترل و سیستم نیم‌سال اول ۱۳۹۹–۱۴۰۰ موعد تحویل: ۹۹/۰۹/۲۷	بنام آنکه جان را فکرت آموخت **کنترل غیرخطی** تمرین سری پنجم	۱۳۰۷ دانشگاه صنعتی خواجه نصیرالدین طوسی استاد: حمید رضا تقی راد

Notice:

- *Attach your codes with the answer.*
- *Any programming language can be used for this question (such as MATLAB, Maple or Python), although the use of MATLAB is recommended.*