



Exercise 1: Consider the following systems:

I. System 1:

$$\dot{x}_1 = -\text{sat}(x_1) + x_1^2 x_2$$

$$\dot{x}_2 = x_1^2 + u$$

II. System 2:

$$\dot{x}_1 = x_1 + x_2$$

$$\dot{x}_2 = \sin(x_1 - x_2) + x_3$$

$$\dot{x}_3 = u$$

Compute a controller using back-stepping to globally stabilize the origin.

Exercise 2: Consider the following system:

$$\dot{x}_1 = x_1^2 \cos(x_1) - \sin(x_1) + x_1 x_2$$

$$\dot{x}_2 = x_3 \cos(x_1) - x_2$$

$$\dot{x}_3 = (x_1 + x_2)^2 + \frac{1}{1 + x_1^2} u$$

Using the back-stepping method, find the state feedback u so that the origin of the system is asymptotically stable and determine the final Lyapunov function of the system.

Exercise 3: Consider the following system:

$$\begin{cases} \dot{x}_1 = x_2 + ax_1 \sin(x_1) \\ \dot{x}_2 = bx_1 x_2 + u \end{cases}$$

Where a, b are uncertainty constant values with the following bounds:

$$|a - 1| \leq 1, \quad |b - 1| \leq 2$$

1. Design a sliding mode controller for global stabilization at the origin.
2. By continuousing this controller using the function $\text{sat}\left(\frac{s}{\varepsilon}\right)$, Examine system stability to prevent chattering within the area $|s| < \varepsilon$ and find an estimate of the upper boundary in such a way that the origin within this region is asymptotically stable.

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Exercise 4: Consider the following system:

$$\dot{x}_1 = x_2 + a \cos(x_1) - x_1$$

$$\dot{x}_2 = -b \sin(x_1) + 4u$$

$$y = x_1$$

Where a is a definite constant value and $1 \leq b \leq 3$. Design the sliding mode controller in such a way that output system follows the continuous and bounded reference $r(t)$ without unwanted oscillations. Determine the gain bound of the sliding mode controller that ensures the robust stability of the closed-loop system.

Exercise 5: Consider following system:

$$\dot{x}_1 = x_2 + \sin(x_1)$$

$$\dot{x}_2 = \theta_1 x_1^2 + (1 + \theta_2)u$$

$$y = x_1$$

where $\theta_1 < 2$ and $\theta_2 < \frac{1}{2}$ using sliding mode control :

1. Design a continuous state feedback controller to stabilize the origin.
2. Design a continuous state feedback controller so that the output $y(t)$ asymptotically tracks a reference signal $r(t)$. Assume that r, \dot{r}, \ddot{r} are continuous and bounded.