استاد: حمید رضا تقی راد	بنام آنکه جان را فکرت آموخت **کنترل غیرخطی** پاسخ آزمون میان ترم	دانشکده مهندسی برق گروه کنترل و سیستم نیمسال اول ۱۳۹۹-۱۴۰۰

PROBLEM 1:

1. The equilibrium point is given by: $x_2^* = 0\,, x_1^* = x_{10}$
2. The Jacobian matrix evaluated at the equilibrium point is given by

$$A = \frac{\partial f(x)}{\partial x} = \begin{bmatrix} 0 & 1 \\ -K_p & -(T_d - 1) - 2|x_2| \end{bmatrix}_{x^*}$$

$$A = \frac{\partial f(x)}{\partial x} = \begin{bmatrix} 0 & 1 \\ -K_p & -(T_d - 1) \end{bmatrix}$$

$$eig(A) = \frac{1}{2}\left(1 - T_d \pm \sqrt{(T_d - 1)^2 - 4K_p}\right)$$

$K_p < \frac{1}{4}(T_d - 1)^2 \rightarrow$ results in two real eigenvalues

$K_p = \frac{1}{4}(T_d - 1)^2 \rightarrow$ results in two equal eigenvalues

$K_p > \frac{1}{4}(T_d - 1)^2 \rightarrow$ results in two complex conjugated eigenvalues

Also, the only possible case for the eigenvalues to have a positive real part is when $T_d < 1$ ($\sqrt{(Td - 1)^2 - 4K_p} < \sqrt{(T_d - 1)^2} = |T_d - 1|$).

3. By using $K_p = T_d = 1$ and $x_{10} = 1$ the system is rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ (1 - x_1) - x_2\,|x_2| \end{bmatrix}$$

Where the equilibrium point is given by:

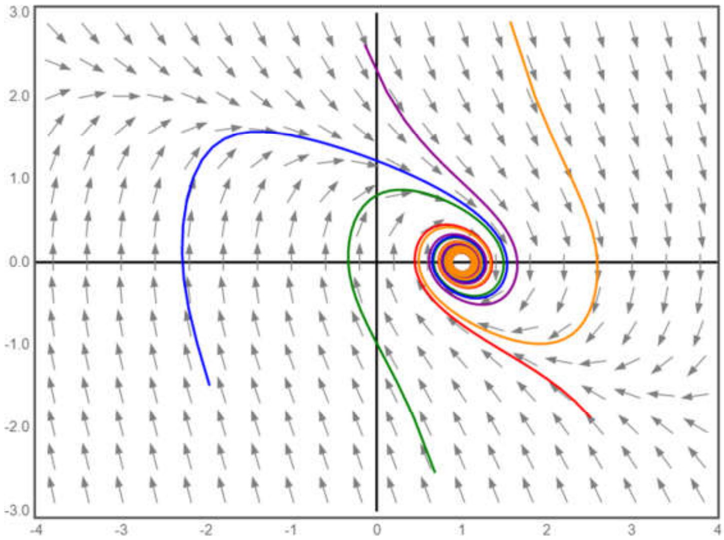
$x_2^* = 0\,, x_1^* = 1$

$eig(A) = \pm j \rightarrow$ *linearization is inconclusive*

Take $x = 1 - x_1$ and $y = x_2$, with $x^* = 0, y^* = 0$,

$V = \frac{1}{2}(x^2 + y^2) \rightarrow \quad \dot{V} = -y^2|y|$ which is negative semi-definite

If $y \equiv 0$ then $\dot{y} = 0$, hence $x = 0$. Therefore the equilibrium point $x_2^* = 0\,, x_1^* = 1$ is stable, and as shown in the phase-plane the eq. point is a stable focus.



PROBLEM 2:

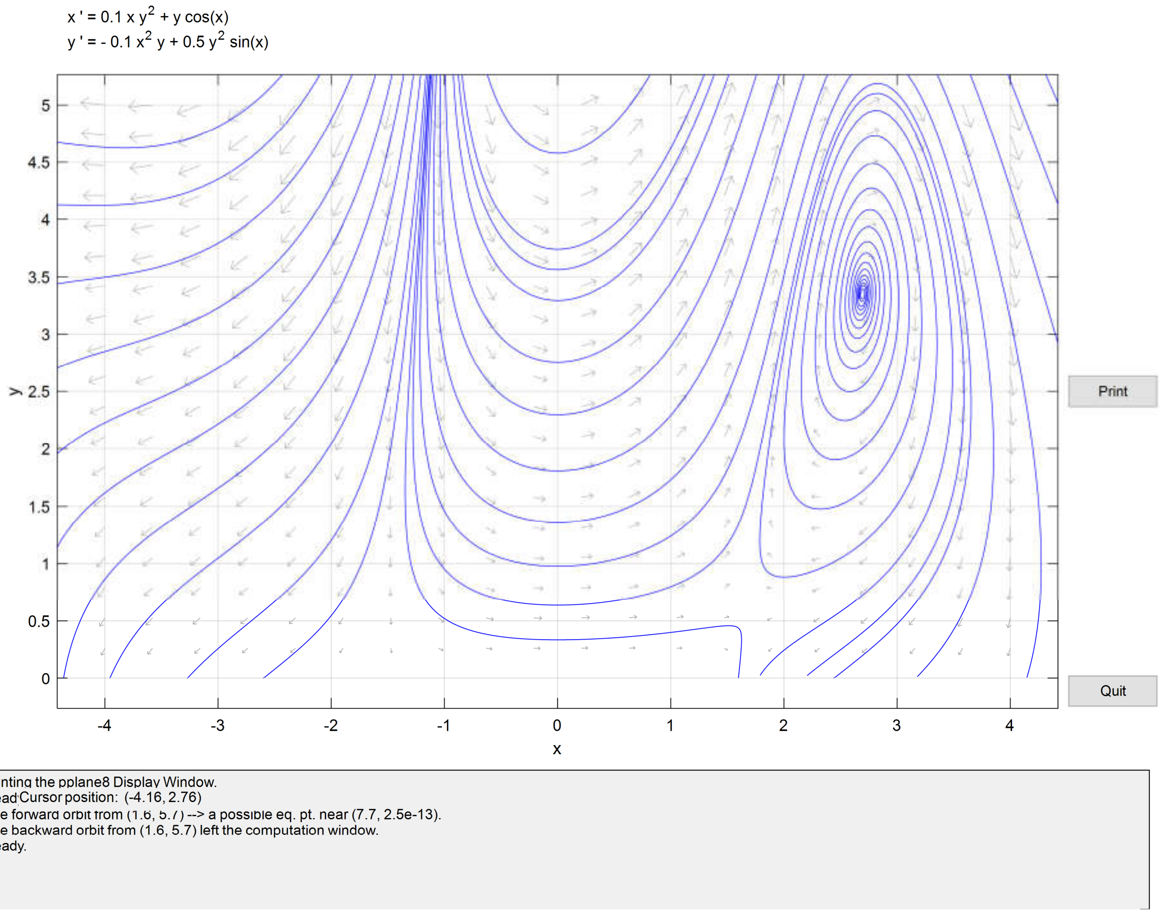
1. Use Bendixon Criteria.

$$\begin{cases} \frac{dx}{dt} = f_1 \\ \frac{dy}{dt} = f_2 \end{cases} \qquad \frac{\partial f_1}{\partial x} = ay^2 - y\sin(x) \quad \& \quad \frac{\partial f_2}{\partial y} = bx^2 + y\sin(x)$$

$$\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} = ay^2 + bx^2$$

For $a, b > 0$ or $a, b < 0$ $\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y}$ doesn't change sign and is not identically zero, the system does not have a limit cycle on the phase plane.

2. For a=0.1, b=-0.1 check the phase portrait:



PROBLEM 3:

$$\dot{V} = 4x_1^3\dot{x}_1 + \frac{\partial\psi(x_2)}{\partial(x_2)}\dot{x}_2 = -4x_1^3\left(\frac{x_2^3}{1+x_2^8}\right) + x_1^3\frac{\partial\psi(x_2)}{\partial(x_2)} - x_2\frac{\partial\psi(x_2)}{\partial(x_2)}$$

$$\dot{V} = x_1^3\left(\frac{\partial\psi(x_2)}{\partial(x_2)} - \left(\frac{4x_2^3}{1+x_2^8}\right)\right) - x_2\frac{\partial\psi(x_2)}{\partial(x_2)}$$

by choosing: $\frac{\partial\psi(x_2)}{\partial(x_2)} = \left(\frac{4x_2^3}{1+x_2^8}\right)$, $\dot{V}$ will be negative-definite. Hence,

$$\psi(x_2) = arc\tan(x_2^4)$$

By this means $V(x)$ is positive-definite. Also $\|x\| \to \infty$ then $(x) \to \infty$, and Lyapunov function $V(x)$ is radially Unbounded. As a result the origin is globally asymptotically stable

PROBLEM 4:

Take $\Delta K = (\widehat{K} - K)$,

$$V(e, \Delta K) = e^T P e + \Delta K^T \Delta K,$$

$$\dot{V} = -e^T Q e \leq 0$$

this inequality implies global uniform stability of the origin and uniform boundedness of $e(t), \Delta K(t)$. Then, because of the system dynamics (A is Hurwitz), $\dot{e}(t)$ is also uniformly bounded, and so the second time derivative of the Lyapunov function

$$\ddot{V} = -2e^T Q \dot{e}$$

is uniformly bounded. Hence applying Barbalat's lemma we have $\left|\left|e(t)\right|\right| \to 0$ as $t \to \infty$.