بنام آنکه جان را فکرت آموخت

کنترل غیرخطی

پاسخ تمرین سری چهارم

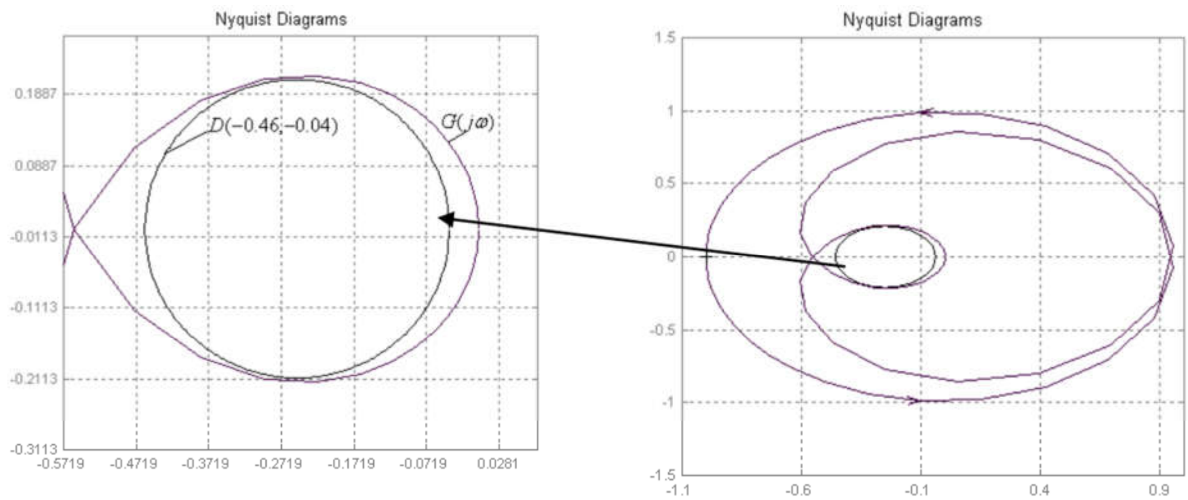
دانشکده مهندسی برق
گروه کنترل و سیستم
نیم‌سال اول ۱۳۹۹–۱۴۰۰
موعد تحویل: ۹۹/۰۹/۱۶

دانشگاه صنعتی خواجه نصیرالدین طوسی ۱۳۰۷
استاد: حمید رضا تقی راد

Solution1:

Because the system has three unstable poles, it must rotate the Disk D(α, β) counterclockwise three times for stability, as shown in the figure below.

Nyquist plot:

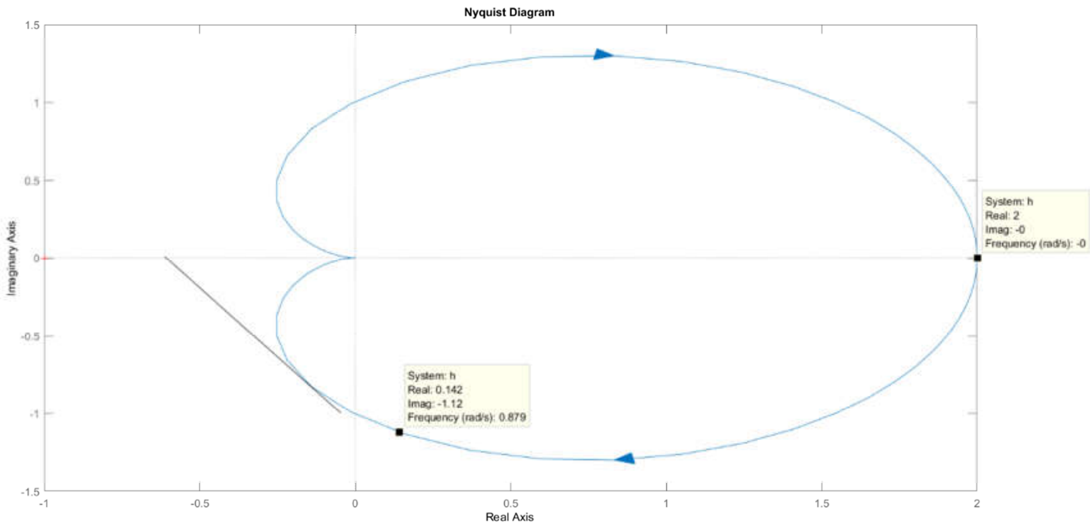


Nyquist with Disk $D(\alpha, \beta)$ Nyquist plot

$$-\frac{1}{\alpha} = -0.46 \ \rightarrow \ \alpha = 2.17 \quad , \quad -\frac{1}{\beta} = -0.04 \ \rightarrow \ \beta = 25$$

Solution2:

The Popov plot is shown in following figure. It is clear from this figure that if $k < 1.66$ then it is always possible to draw a straight line through the point $\frac{-1}{k}$ such that the plot lies to the right of the straight line Hence, by Popov criteria, we conclude that the feedback system is globally asymptotically stable for all time-invariant nonlinearities in the sector (0,1.66).



Solution3:

1. When the saturation works in the linear range, we have the closed loop dynamics

$$G(s) = \frac{-5s}{s^2 + (1 - 5H)s + 25}$$

which is unstable for H > 0.2. Thus, the state cannot remain small. In saturation, on the other hand, the nonlinearity generates a constant (“step”) input to the system. The final value theorem then gives

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} \frac{-5s}{s^2 + s + 25} = 0$$

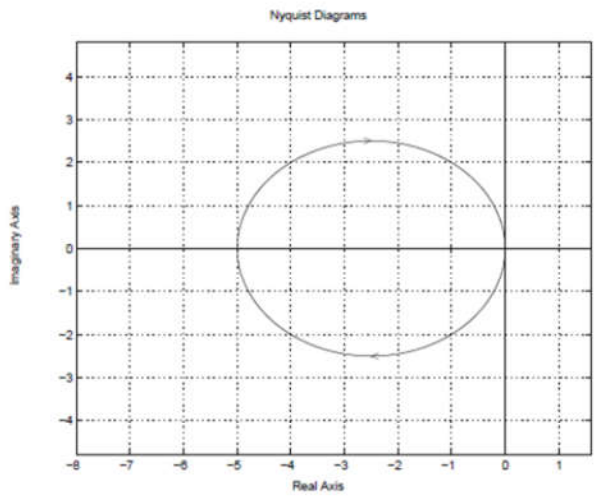
The observation that $y(t) \to 0$ contradicts the assumption that the nonlinearity remains saturated.

2. We should investigate intersection of the Nyquist curve and $-1/A$. Since $A \in [H, \infty)$, $-1/N(A)$ lies in the interval $(-\infty, -1/H)$.

$$G(i\omega) = \frac{-i5\omega}{25 - \omega^2 + i\omega}$$

which intersects the negative real axis for $\omega' = 5\, rad/s$. The value of $G(i\omega') = -5$. Thus, there will be an intersection if $H > 0.2$. The frequency of the oscillation is estimated to 5 rad/s, and for fixed H.

3. The Nyquist curve of the system is shown in the following figure. The function $-1/N(A)$ is also displayed, with an arrow in the direction of increasing A. The Nyquist curve encircles the points $Re(G(i\omega)) > -5$, indicating increased oscillation amplitude. The points to the left of the intersection are not encircled, indicating stability and a decaying oscillation amplitude. We can thus expect a stable limit cycle.



Nyquist curve and $-1/N(A)$ for oscillator example.

Solution4:

1. Saturation: Consider the input $x(t) = Asin(wt)$

$$N(a) = \frac{2k}{\pi}\left[\sin^{-1}\frac{\delta}{a} + \frac{\delta}{a}\sqrt{1-\left(\frac{\delta}{a}\right)^2}\right]$$

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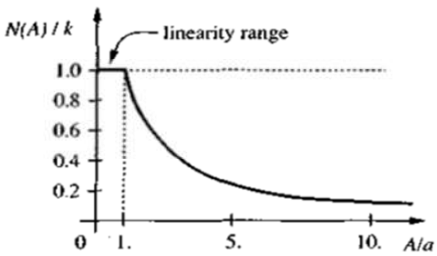
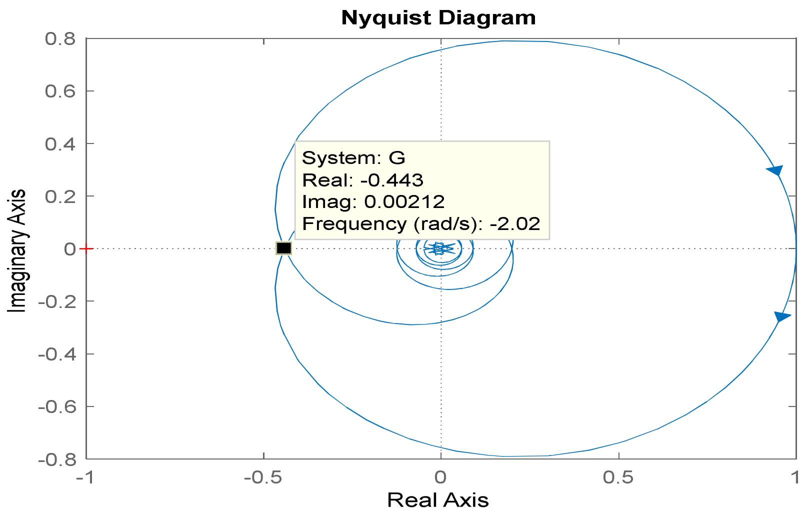
✓ D.F. Features:

1. $N(A) = k$ if the input amplitude is in the linearity range
2. $N(A)$ decreases as the input amplitude increases
3. there is no phase shift

✓ Graphical Representation



✓ Intersect $G(j\omega)$ and $-1/N(A)$
✓ If intersection occurs ⇨ L.C. exists
✓ *Amplitude* of L.C. ⇨ A (on $-1/N(A)$)
✓ *frequency* of L.C. ⇨ ω (on $G(j\omega)$)



$$k = \frac{1}{0.443} = 2.257$$

According to the above Nyquist diagram as well as the Nyquist saturation curve, for k> 2.257, two Nyquists intersect and a boundary cycle occurs.

Stability: The Nyquist diagram -1/N(a) moves from the instability zone to the stability zone as a increases. Therefore, the cycle is somewhat stable.

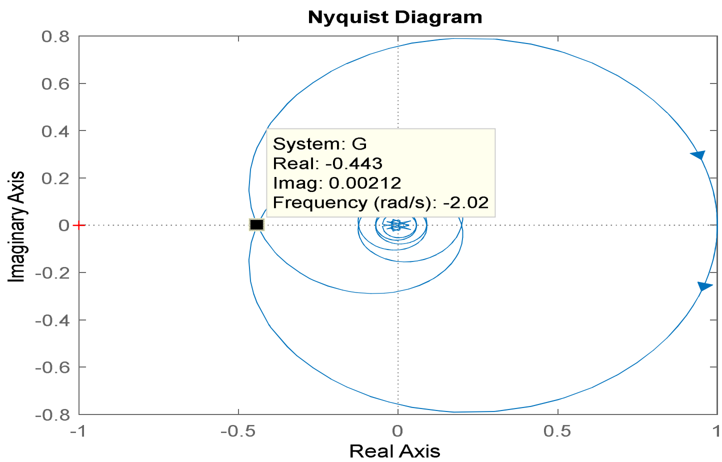
2.

$$G(jw) = \frac{e^{-jw}}{jw+1} \times \frac{-jw+1}{-jw+1}$$

$$G(jw) = \frac{(1-jw)(\cos w - j\sin w)}{w^2+1}$$

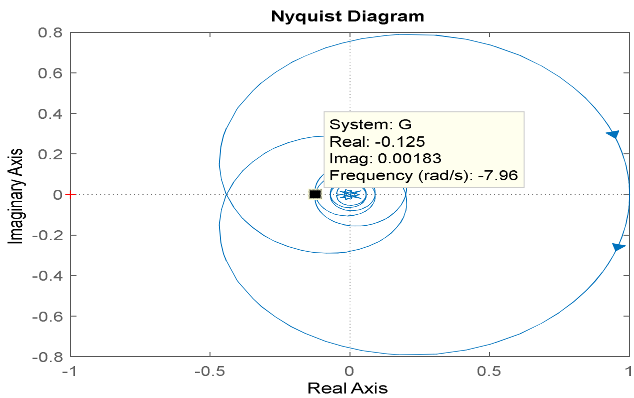
$$G(jw) = \frac{1}{w^2+1}[(\cos w - w\sin w) - j(\sin w + w\cos w)]$$

$$Im\left(G(jw)\right) = 0 \quad \Longrightarrow \sin w + w\cos w = 0 \quad \Longrightarrow \tan w = -w$$



According to the Nyquist diagram as well as the Nyquist saturation curve, the two Nyquists intersect at the frequency w = 2.02 and a limit cycle is created.

3. According to the following figure, assuming k = 5, the Nyquist curve -1/N(a) starts from -0.2 and tends to $-\infty$.



Also, according to the saturation curve, the value is $\delta = 2$. However, the intersection of two curves occurs at only one point. The value of k is calculated in part 1 for this point. Therefore, according to the following curve of the cycle, amplitude limit of 5.52 is considered.