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Adaptive control of parallel robots with uncertain kinematics and dynamics

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One of the most challenging issues in adaptive control of robot manipulators with kinematic uncertainties is the requirement of inverse Jacobian matrix in regressor form. This requirement is inevitable in the case of the control of parallel robots, whose dynamics formulation are derived in the task space. In this paper, an adaptive controller is proposed for parallel robots based on representation of Jacobian matrix in regressor form with asymptotic trajectory tracking. The main idea of this paper is separation of determinant and adjugate of Jacobian matrix in order to represent them into a new regressor forms. Simulation and experimental results on a 2-DOF R$\underline{\text{P}}$R and a 3-DOF redundant cable driven robot, respectively, verify promising performance of the proposed method in practice.

1. Introduction

Uncertainties in dynamic and kinematic parameters are inseparable part of robotic systems [1]. To design effective controllers in presence of uncertainty, several methods are reported in the literature [2,3]. One of the powerful methods is adaptive control [4]. Adaptive controllers are developed to dispel dynamic uncertainties in both serial [5] and parallel robots [6], and furthermore, in bilateral teleoperation manipulators [7]. The main idea in this method is to express dynamic formulation in regressor form, and furthermore, to derive an adaptation law for unknown parameters based on Lyapunov analysis [8]. In this regard, the first Jacobian adaptation algorithm for serial robots was presented in [9], where the velocity equations of the robot was expressed in regressor form with respect to unknown kinematic parameters. By using Lyapunov direct method, it is shown that task space variables track the desired trajectory, whereas parameters estimations may not necessarily converge to their real values [10,11]. Note that in the last two works, it is assumed that an equation containing inverse of Jacobian matrix can be expressed in regressor form. Wang in [12] resolve this problem and proposed new adaptation laws which improved the performance of the closed-loop system. However, these works are focused on serial robots, and less attention has been paid to the control of parallel robots with kinematic uncertainties.

Parallel robots are closed–loop mechanisms in which the moving platform is linked to the base by several independent kinematic chains [13,14]. The unique characteristics of parallel robots in terms of their speed and rigidity make them suitable to a variety of applications such as flight simulators and very fast pick and place manipulators [15]. Cable driven parallel

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robots are a prominent class of this robots where the links are formed by cables driven by actuators [16,17]. Since dynamics formulation of these robots are usually written in task space [18], exact values of dynamic parameters and Jacobian matrix is required to achieve a precise trajectory tracking. This condition may not be satisfied in most cases especially for deployable Cable Driven Robots (CDRs), where a calibrated model is usually unavailable [19,20].

Although calibration methods are well developed to reduce kinematic uncertainties [21], they usually does not overcome Jacobian uncertainties and are not applicable for special cases such as deployable CDRs. Another crucial issue in large scale CDRs is sagging of cables [22]. In this situation, the kinematic and Jacobian matrix are changed based on position of end-effector, and therefore, a control strategy to adapt kinematic and Jacobian is strictly required. Authors in [23,24] have taken two approaches to tackle this problem for a specified robot. In [23], an adaptive controller is proposed, and it is assumed that the adapted parameters converge to their physical values. This assumption is not necessarily fulfilled in practice since there is no theoretical guarantee for such convergence. In [24] an adaptive robust controller is proposed, in which the bounds of dynamic and kinematic estimation errors are considered to be constant but unknown and at last an ultimate bound for tracking error is derived. However, since these bounds are state-dependent, this assumption may not be easily fulfilled.

In this paper an adaptive controller based on [8], is developed for parallel manipulators with kinematic and dynamic uncertainties. In contrast to the majority of works reported in the literature which are merely focused on fully parallel case, the proposed method works well for both fully and redundantly actuated robots. Invoking the researches in the field of serial robots, the main contribution of this paper is based on a novel representation of Jacobian matrix of the robot in a general regression form, i.e. instead of expressing velocity terms in regressor form, the Jacobian matrix is represented in regressor form which clearly results in a matrix of unknown values. In order to determine an expression for the inverse Jacobian matrix in regressor form, which is necessary but computationally challenging part of control law, we propose to separate adjugate and determinant of this matrix and then form new regressors. By this means and based on passivity method, trajectory tracking is achievable and is analyzed using direct Lyapunov method. One of the main advantages of this method is that it relaxes the assumption that an equation containing inverse of Jacobian matrix should be expressed in regressor form. Hence, the scope of application of this paper is much wider in practice. To the best of the authors' knowledge, such controller synthesis is not fully addressed before in the field of parallel robots with detailed analysis.

Notation: For any matrix $A \in \mathbb{R}^{n\times m}$, A_i denotes i-th column, ${}_jA$ denotes j-th row and $A_{i,j}$ denotes (i,j)-th element of A. A^* and $A^\dagger$ represent adjugate and right pseudo-inverse of A, respectively, while $\widehat{A}$ represents an estimate value of A while $\widetilde{A} = \widehat{A} - A$. Unless indicated otherwise, all vectors in the paper are considered as column vectors.

2. Kinematics and dynamics analysis

The dynamic model of a parallel robot with n degrees of freedom and m actuators with negligible dissipation forces may be written in the task space as follows [19]:

$$M(X)\ddot{X} + C\left(X,\dot{X}\right)\dot{X} + G(X) + d = F = J^T(X)\tau, \tag{1}$$

where $X, \dot{X} \in \mathbb{R}^n$ denotes the generalized coordinate vector representing the position and orientation of the end–effector and their velocities, respectively, $\tau \in \mathbb{R}^m$ denotes the applied torque to the robot, $M(X) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C\left(X,\dot{X}\right) \in \mathbb{R}^{n\times n}$ denotes the Coriolis and centrifugal matrix, $G(X) \in \mathbb{R}^n$ is the vector of gravity terms, $J(X) \in \mathbb{R}^{m\times n}$ denotes the Jacobian matrix of the robot and $d \in \mathbb{R}^n$ denotes the external disturbance applied to the moving platform which is assumed to be bounded. Some important properties of the robot dynamic formulation (1) from [[18], Sec. 5.5.4] are as follows.

P1: The inertia matrix $M(X)$ is symmetric and positive definite for all X.

P2: The matrix $\dot{M}(X) - 2C\left(X,\dot{X}\right)$ is skew symmetric.

P3: The dynamic model is linear with respect to a set of dynamical parameters and may be represented in a linear regression form:

$$M(X)\ddot{X} + C\left(X,\dot{X}\right)\dot{X} + G(X) = Y_m\left(\ddot{X},\dot{X},X\right)\theta_m, \tag{2}$$

where, $Y_m\left(\ddot{X},\dot{X},X\right)$ denotes the regressor matrix and θ_m denotes the dynamic parameters vector.

The task space wrench F is related to joint space force vector τ by Jacobian transpose:

$$F = J^T(X)\tau. \tag{3}$$

It was shown in [9] that for serial robots, the Jacobian matrix may be expressed in regressor form as:

$$J(q)\dot{q} = Y_k(q,\dot{q})\theta_k, \tag{4}$$

where θ_k denotes unknown kinematic parameters in Jacobian matrix. This expression may be used to represent each element of the Jacobian matrix as a linear regression form of kinematic parameters as:

$$J_{i,j}^T(q) = Y_{k_i}\theta_{k_j}, \qquad Y_{k_i} \in \mathbb{R}^{1\times l_{i,j}}, \quad \theta_{k_j} \in \mathbb{R}^{l_{i,j}\times 1}, \tag{5}$$

where the vectors Y_{k_i} and θ_{k_j} represent the known and unknown variables, respectively. Since a matrix may be represented by its elements as follows

$$J^T = \sum_{i=1}^{n}\sum_{j=1}^{m} J_{i,j}^T\,{}^i\Psi^j, \qquad {}^i\Psi^j \in \mathbb{R}^{n\times m},$$

where all elements of ${}^i\Psi^j$ are zero except (i,j)-th element which is equal to one. Hence, one may represent $J^T(q)$ as follows:

$$J^T(q) = \sum_{i=1}^{n}\sum_{j=1}^{m} Y_{k_i}\theta_{k_j}\,{}^i\Psi^j. \tag{6}$$

In parallel robots with actuated revolute joints, Jacobian matrix may be expressed in the form of (6). However, Jacobian matrix of a general 6-DOF actuated prismatic joints robot including cable driven robots may be represented by, [18, Ch.4]:

$$J(X) = \begin{bmatrix} \hat{\lambda}_1^T & \left({}^bR_p a_1 \times \hat{\lambda}_1\right)^T \\ \vdots & \vdots \\ \hat{\lambda}_m^T & \left({}^bR_p a_m \times \hat{\lambda}_m\right)^T \end{bmatrix}, \qquad \hat{\lambda}_i, a_i \in \mathbb{R}^3, \tag{7}$$

where $\hat{\lambda}_i$ denotes unit vector in the link direction, a_i denotes the attachment points of the links to the end-effector represented in moving frame and ${}^bR_p$ denotes the rotation matrix. On the contrary, it is not straight forward for these manipulators to express $J(X)$ in the form of (6) due to fractional elements of the matrix. To overcome this problem, $J^T(X)$ is expressed in the following form:

$$J^T = \begin{bmatrix} \lambda_1 & \dots & \lambda_m \\ \left({}^bR_p a_1 \times \lambda_1\right) & \dots & \left({}^bR_p a_m \times \lambda_m\right) \end{bmatrix} \begin{bmatrix} l_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & l_m \end{bmatrix}^{-1} \triangleq J_{new}^T(X)L^{-1}, \tag{8}$$

where $\lambda_i = l_i\hat{\lambda}_i$ and l_i as the length of i-th link. Through this formulation, it is possible to define $J_{new}^T(X) \in \mathbb{R}^{n\times m}$ in the regressor form of (6). Invoking (6), we assume that $J_{new}^T(X)$ is expressed in the following compact form

$$J_{new}^T(X) = Y(X)\Theta, \tag{9}$$

in which $Y(X) \in \mathbb{R}^{n\times l}$ and $\Theta \in \mathbb{R}^{l\times m}$. Notice that it is possible to show that this formulation is not limited to merely the Jacobian matrices represented by (7).

3. Adaptive Jacobian controller

In this section an adaptive controller based on [8], is proposed for a parallel manipulator with uncertain kinematics and dynamics. It is assumed that position and velocity of end-effector, as well as the length of links are available for feedback and Jacobian matrix is expressed in the form of (9). In the proposed controller, trajectory tracking is guaranteed by augmentation of the proposed controller in [8], and adaptation law for the unknown parameters.

Let us define S as in [24,8]:

$$S = \dot{\tilde{X}} + \Gamma\tilde{X} = \dot{X} - \dot{X}_r, \tag{10}$$

with

$$\tilde{X} = X - X_d, \qquad \dot{X}_r = \dot{X}_d - \Gamma\tilde{X}, \tag{11}$$

where $X_d \in \mathscr{C}^2$ denotes the desired trajectory, $X_r = X_d - \Gamma\int_0^t \tilde{X}\,dt$ denotes virtual reference trajectory and Γ is a constant positive definite matrix. If all the kinematics and dynamics parameters are known, the following control law may be directly used to achieve a suitable performance

$$\tau = LJ_{new}^{\dagger}\left(M\ddot{X}_r + C\dot{X}_r + G - KS - K'\mathrm{sign}(S)\right), \tag{12}$$

in which, K and K' are constant symmetric positive definite matrices, and $J_{new}^{\dagger}$ denotes the right pseudo-inverse of $J_{new}^{T}(X)$. In the case of fully actuated robots, $J_{new}^{\dagger}$ is replaced by J_{new}^{-T}. Note that control law (12) is related to Jacobian matrix in the form (8), while for actuated revolute joint, the control law is

$$\tau = J^{\dagger}\left(M\ddot{X}_r + C\dot{X}_r + G - KS - K'\mathrm{sign}(S)\right).$$

In the sequel, continue with the notation (12). Invoking [25, Ch.1], $J_{new}^{\dagger}$ is given as

$$J_{new}^{\dagger} \triangleq \frac{R}{T},$$

in which for the case of redundantly actuated robot where $J_{new}^{\dagger} = J_{new}\left(J_{new}^{T}J_{new}\right)^{-1}$, R and T are derived as

$$R = J_{new}\left(J_{new}^{T}J_{new}\right)^{*} \in \mathbb{R}^{m\times n}, T = \det\left(J_{new}^{T}J_{new}\right) \in \mathbb{R}, \tag{13}$$

and for the case of fully parallel robots, i.e. the robots with the number of actuators equal to degrees of freedom,

$$R = \left(J_{new}^{T}\right)^{*} \in \mathbb{R}^{n\times n}, T = \det\left(J_{new}^{T}\right) \in \mathbb{R}, \tag{14}$$

where $(\cdot)^{*}$ denotes the adjugate matrix. Due to the uncertainties in parameters, we have to use the estimated values in the control law. Thus,

$$\tau = L\frac{\widehat{R}}{\widehat{T}}\left(\widehat{M}\ddot{X}_r + \widehat{C}\dot{X}_r + \widehat{G} - KS - K'\mathrm{sign}(S)\right), \tag{15}$$

where, $(\hat{\cdot})$ denotes the estimated value. Invoking **P3** and the fact that adjugate matrix is linear with respect to the parameters, assume that it is possible to write:

$$\widehat{R}\left(\widehat{M}\ddot{X}_r + \widehat{C}\dot{X}_r + \widehat{G} - KS - K'\mathrm{sign}(S)\right) = Y_a\left(X,\dot{X},\dot{X}_r,\ddot{X}_r\right)\hat{\theta}_a, \tag{16}$$

where $\theta_a \in \mathbb{R}^r$ is constructed by concatenation of kinematics and dynamics parameters into a vector. Replace (8), (15) and (16) in (1), the closed-loop dynamic may be written as:

$$M(X)\ddot{X} + C\left(X,\dot{X}\right)\dot{X} + G(X) + d = J_{new}^{T}\frac{Y_a\hat{\theta}_a}{\widehat{T}}. \tag{17}$$

Add

$$-J_{new}^{T}\frac{Y_a\theta_a}{\widehat{T}}$$

to both sides of (17), to obtain:

$$M(X)\ddot{X} + C\left(X,\dot{X}\right)\dot{X} + G(X) + d - \frac{T}{\widehat{T}}\left(M\ddot{X}_r + C\dot{X}_r + G - KS - K'\mathrm{sign}(S)\right) = J_{new}^{T}(X)\frac{Y_a\tilde{\theta}_a}{\widehat{T}}, \tag{18}$$

where $\tilde{\theta}_a = \hat{\theta}_a - \theta_a$ denotes estimation error. Notice that the determinant of a matrix is linear with respect to the elements of that matrix. Thus we may express T as a linear regression $T = Y_b(X)\theta_b$ where $\theta_b \in \mathbb{R}^k$ are unknown parameters in the determinant. On the other hand, considering **P3**, one may reach to the following equation:

$$M\ddot{X}_r + C\dot{X}_r + G - KS - K'\mathrm{sign}(S) = Y_c\left(X,\dot{X},\dot{X}_r,\ddot{X}_r\right)\theta_c, \tag{19}$$

where $\theta_c \in \mathbb{R}^p$ denotes the vector of dynamical parameters. Using (19), left hand side of (18) is rewritten as follows:

$$\begin{aligned} &M(X)\ddot{X} + C\left(X,\dot{X}\right)\dot{X} + G(X) + d - \frac{Y_b\left(\hat{\theta}_b - \tilde{\theta}_b\right)}{Y_b\hat{\theta}_b}\left(M\ddot{X}_r + C\dot{X}_r + G - KS - K'\mathrm{sign}(S)\right) \\ &= M\dot{S} + CS + KS + K'\mathrm{sign}(S) + d + \frac{Y_b\tilde{\theta}_b}{Y_b\hat{\theta}_b}\left(M\ddot{X}_r + C\dot{X}_r + G - KS - K'\mathrm{sign}(S)\right) \\ &= M\dot{S} + CS + KS + K'\mathrm{sign}(S) + d + \frac{Y_b\tilde{\theta}_b}{Y_b\hat{\theta}_b}Y_c\theta_c. \end{aligned} \tag{20}$$

where we used the relations $T = Y_b\theta_b = Y_b\left(\hat{\theta}_b - \tilde{\theta}_b\right), \widehat{T} = Y_b\hat{\theta}_b$ and

$$M(X)\ddot{X}+C\left(X,\dot{X}\right)\dot{X}+G(X)-\left(M\ddot{X}_r+C\dot{X}_r+G-KS-K'\mathrm{sign}(S)\right)=M\dot{S}+CS+KS+K'\mathrm{sign}(S).$$

Finally, using (9), closed-loop Eq. (18) is reduced to:

$$M\dot{S}+CS+d+KS+K'\mathrm{sign}(S)=Y\hat{\Theta}\frac{Y_a\tilde{\theta}_a}{Y_b\hat{\theta}_b}-Y\tilde{\Theta}\frac{Y_a\tilde{\theta}_a}{Y_b\hat{\theta}_b}+\frac{Y_b\tilde{\theta}_b}{Y_b\hat{\theta}_b}Y_c\tilde{\theta}_c-\frac{Y_b\tilde{\theta}_b}{Y_b\hat{\theta}_b}Y_c\hat{\theta}_c, \tag{21}$$

in which $J_{new}^T=Y\hat{\Theta}-Y\tilde{\Theta}$ and $Y_c\theta_c=Y_c\hat{\theta}_c-Y_c\tilde{\theta}_c$ are utilized. Now let us design adaptation laws based on passivity method. For this means, recall Proposition 4.3.1 of [26] on the connection of two passive systems. The reader is referred to this reference for detailed proof.

Proposition 1. [26]. Consider the standard feedback of systems Σ_1 and Σ_2 which is shown in Fig. 1. Assume that Σ_1 is output strictly passive, i.e. there exists a storage function H_1 such that $\dot{H}_1\leqslant u_1^Ty_1-y_1\psi(y_1)$ where $y_1\psi(y_1)\geqslant 0$, and Σ_2 is passive, i.e. there exists a storage function H_2 such that $\dot{H}_2\leqslant u_2^Ty_2$. Then the states of Σ_1 converge to zero while states of Σ_2 remain bounded.□

In order to use the above proposition, we shall modify right hand side of (21) in such a way that all the terms are represented by a regressor matrix and an unknown vector. Therefore, in the following, $Y\tilde{\Theta}\frac{Y_a\tilde{\theta}_a}{Y_b\hat{\theta}_b}$ and $\frac{Y_b\tilde{\theta}_b}{Y_b\hat{\theta}_b}Y_c\tilde{\theta}_c$ are changed accordingly. Assume that

$$\frac{Y_b\tilde{\theta}_b}{Y_b\hat{\theta}_b}Y_c\tilde{\theta}_c\triangleq\frac{Y_cY_\mu}{Y_b\hat{\theta}_b}\tilde{\theta}_\mu,\quad\text{with}\quad Y_\mu=\begin{bmatrix}Y_b & 0_{1\times k} & \dots & 0_{1\times k}\\ 0_{1\times k} & Y_b & 0_{1\times k} & \dots\\ \vdots & & \ddots & \vdots\\ 0_{1\times k} & \dots & 0_{1\times k} & Y_b\end{bmatrix}_{n\times p\cdot k},\qquad \tilde{\theta}_\mu=\begin{bmatrix}\left({}_1\left(\tilde{\theta}_c\tilde{\theta}_b^T\right)\right)^T\\ \vdots\\ \left({}_p\left(\tilde{\theta}_c\tilde{\theta}_b^T\right)\right)^T\end{bmatrix}$$

where ${}_i\left(\tilde{\theta}_c\tilde{\theta}_b^T\right)$ is the i-th row of $\tilde{\theta}_c\tilde{\theta}_b^T$. Furthermore,

$$Y\tilde{\Theta}\frac{Y_a\tilde{\theta}_a}{Y_b\hat{\theta}_b}\triangleq\frac{YY_\eta}{Y_b\hat{\theta}_b}\tilde{\theta}_\eta,\quad\text{with}\quad Y_\eta=\begin{bmatrix}{}_1(Y_a),\dots,{}_m(Y_a) & 0_{1\times m.r} & \dots & 0_{1\times m\cdot r}\\ 0_{1\times m\cdot r} & {}_1(Y_a),\dots,{}_m(Y_a) & & 0_{1\times m\cdot r}\\ \vdots & & \ddots & \vdots\\ 0_{1\times m\cdot r} & \dots & & {}_1(Y_a),\dots,{}_m(Y_a)\end{bmatrix}_{l\times m\cdot r\cdot l},$$

in which ${}_i(Y_a)$ is i-th row of Y_a, and

$$\tilde{\theta}'_\eta=\begin{bmatrix}\tilde{\Theta}_{1,1}\tilde{\theta}_a^T & \dots & \tilde{\Theta}_{1,m}\tilde{\theta}_a^T\\ \vdots & \ddots & \vdots\\ \tilde{\Theta}_{l,1}\tilde{\theta}_a^T & \dots & \tilde{\Theta}_{l,m}\tilde{\theta}_a^T\end{bmatrix}_{l\times m\cdot r},\qquad \tilde{\theta}_\eta=\begin{bmatrix}{}_1\left(\tilde{\theta}'_\eta\right)^T\\ \vdots\\ {}_l\left(\tilde{\theta}'_\eta\right)^T\end{bmatrix}_{m\cdot r\cdot l\times 1}$$

where $\tilde{\Theta}_{i,j}$ is (i,j)-th element of $\tilde{\Theta}$. Therefore, (21) may be rewritten as follows:

$$M\dot{S}+CS+d+KS+K'\mathrm{sign}(S)=\frac{Y\hat{\Theta}Y_a}{Y_b\hat{\theta}_b}\tilde{\theta}_a-\frac{YY_\eta}{Y_b\hat{\theta}_b}\tilde{\theta}_\eta+\frac{Y_cY_\mu}{Y_b\hat{\theta}_b}\tilde{\theta}_\mu-\frac{Y_c\hat{\theta}_cY_b}{Y_b\hat{\theta}_b}\tilde{\theta}_b\triangleq Y_F\tilde{\theta}_F, \tag{22}$$

with

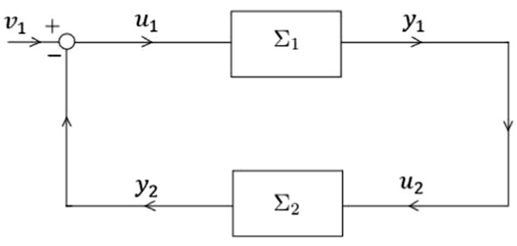


Fig. 1. Standard feedback configuration of two systems.

$$Y_F = \begin{bmatrix} \frac{Y\hat{\Theta}Y_a}{Y_b\hat{\theta}_b} & -\frac{YY_\eta}{Y_b\hat{\theta}_b} & \frac{Y_cY_\mu}{Y_b\hat{\theta}_b} & -\frac{Y_c\hat{\theta}_cY_b}{Y_b\hat{\theta}_b} \end{bmatrix}, \qquad \tilde{\theta}_F = \begin{bmatrix} \tilde{\theta}_a \\ \tilde{\theta}_\eta \\ \tilde{\theta}_\mu \\ \tilde{\theta}_b \end{bmatrix} \in \mathbb{R}^{(r+m\cdot r\cdot l+p\cdot k+k)\times 1}$$

in which Y_F and θ_F are regressor and the vector of all of the uncertain parameters, respectively. Eq. (22) may be considered as the system Σ_1 represented in the mentioned proposition. This system is output strictly passive with $H_1 = \frac{1}{2}S^TMS$ as the storage function, because

$$\dot{H} \leqslant -\lambda_{min}\{K\}\|S\|^2 - (\lambda_{min}\{K'\} - \rho)\|S\| + S^Tu_1, \qquad \text{where}\|d\| \leqslant \rho,$$

with $u_1 = Y_F\tilde{\theta}_F, y_1 = u_2 = S$ and without external input (i.e. $v_1 = 0$). In order to apply the aforesaid proposition, an adaptation law is required to be defined for $\tilde{\theta}_F$ such that Σ_2 becomes passive. For this means, the following dynamic is set for $\tilde{\theta}_F$

$$\dot{\tilde{\theta}}_F = -\Lambda^{-1}Y_F^TS \tag{23}$$

with $\Lambda > 0$ as the gain of adaptation law which leads to the passivity of Σ_2, since

$$H_2 = \frac{1}{2}\tilde{\theta}_F^T\Lambda\tilde{\theta}_F \rightarrow \dot{H}_2 = -S^TY_F\tilde{\theta}_F = u_2^Ty_2.$$

Note that it is assumed that θ_F is constant. Let us state the following theorem on adaptive control of parallel robots with kinematics and dynamics uncertainties.

Theorem 1. *Consider a parallel robot with dynamic Eq. (1). Assume that (9) and (16) hold, $\|d\| \leqslant \rho$ and control and adaptation laws are given as in (15) and (23) with $\lambda_{min}\{K'\} > \rho$, respectively. Then, the closed-loop tracking error converges to zero in the presence of uncertainties in kinematics and dynamics parameters.*

Proof. The proof is obvious with respect to the mentioned proposition and Σ_1, Σ_2 defined above. However, a Lyapunov based proof is also presented here. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}S^TMS + \frac{1}{2}\tilde{\theta}_F^T\Lambda\tilde{\theta}_F. \tag{24}$$

Then its time derivative becomes

$$\dot{V} \leqslant -S^TKS - (\lambda_{min}\{K'\} - \rho)\|S\|.$$

Invoking Lasalle-Yoshizawa Theorem [27, Theorem 8.4], it is easy to show that S converges to zero, and hence, the convergence of $\tilde{X}$ is resulted from (10). ■.

Remark 1. One of the advantages of the proposed method compared to previous works on this topic is that the closed loop equation was expressed in the form of (22) which reminiscent the closed loop equation of well-known adaptive controller proposed in [8]. Hence, for both kinematics and dynamics parameters merely one adaptation law was proposed with the expense of over parameterization. Note that Eqs. (9) and (16) are gentle form of what is proposed in [10,11], where it is assumed that a term containing $\hat{J}^{-1}$ is expressed in regressor form.

Remark 2. Singularity avoidance in construction and path planing is a necessary and important requirement in parallel robots [28,29]. Here, it is assumed that desired trajectory is inside the workspace and away from singular configurations of the robot. By this means, the Jacobian matrix is always of full rank, and therefore, it is possible to find its estimate. However, projection algorithm may be employed in order to ensures singularity avoidance as well as avoiding large variation in parameters and provides a faster and better transient response. Note that by this means, positive tension in the case of CDRs is ensured.

In the following lemma, invoking [4, Theorem 4.4.1], a projection algorithm based on gradient method is proposed.

Lemma 1. *Consider closed-loop system (22) with adaptation law (23). Assume that it is priori known that θ_F is absolutely in a compact subspace Ω, i.e. $\theta_F \in \Omega$ where Ω is defined as $\Omega = \{\theta_F | g(\theta_F) \leqslant 0\}$ and $g(\theta_F)$ is known. The objective is to keep $\hat{\theta}_F$ in Ω. If $\hat{\theta}_F$ is on the edge of Ω i.e. $\hat{\theta}_F \in \partial\Omega$, and $\dot{\hat{\theta}}_F^T\nabla g > 0$, the following adaptation law is chosen*

$$\dot{\hat{\theta}}_F = -\Lambda^{-1}\overline{\nabla g}Y_F^TS \tag{25}$$

where $\overline{\nabla g}$ is projection matrix

$$\overline{\nabla g} = I - \frac{(\nabla g)(\nabla g)^T}{||\nabla g||^2}. \tag{26}$$

This leads to $\hat{\theta}_F$ to remains in Ω.

Proof. If $\hat{\theta}_F$ is inside Ω, adaptation law (23) is applied, and by this means, it remains in Ω. Assume that $\hat{\theta}_F \in \partial\Omega$, hence the aim is to ensure that $\hat{\theta}_F$ always remains in Ω. For this means, the direction of $\dot{\hat{\theta}}_F$ should not be directed toward outside of Ω. In other words, dot product of $\dot{\hat{\theta}}_F$ and $\nabla g = \partial g/\partial\theta$ shall be non-positive. Therefore, if $\left(-\Lambda^{-1}Y_F^TS\right)^T\nabla g > 0, \dot{\hat{\theta}}_F$ should be projected on the direction tangent to $\partial\Omega$. This is done using projection matrix (26) which results in adaptation law (25). Now consider Lyapunov candidate (24) whose time derivative is given as:

$$\dot{V} = S^Td - S^TKS - S^TK'\text{sign}(S) + S^TY_F\tilde{\theta}_F - \tilde{\theta}_F^T\overline{\nabla g}Y_F^TS.$$

By considering (26), $\dot{V}$ becomes

$$\dot{V} \leqslant -\lambda_{min}\{K\}||S||^2 - (\lambda_{min}\{K'\} - \rho)||S|| + \tilde{\theta}_F^T\frac{(\nabla g)(\nabla g)^T}{||\nabla g||^2}Y_F^TS.$$

Note that $\tilde{\theta}_F^T\nabla g \geqslant 0$, since direction of $\tilde{\theta}_F$ and ∇g are toward outside of Ω. Therefore, the last term in the above inequality is negative. ■.

Notice that in most cases, exact derivation of $g(\theta_F)$ is highly complicated. Hence, the acceptable bound for each element of unknown vector is considered and the simplest projection function, namely saturation is used, since it is applicable to any adaptive control law [30]. In other words, this is equivalent to define an absolute value function for every elements of unknown vectors. For example, Assume that $\alpha \in \mathbb{R}$ is an element of an unknown vector and it is known that $k_1 \leqslant \alpha \leqslant k_2$. Define $g(\alpha)$ as

$$g(\alpha) = \left|\alpha - \frac{k_1 + k_2}{2}\right| - \frac{k_2 - k_1}{2}.$$

Now, one may find $\nabla g = \text{sign}\left(\alpha - \frac{k_1+k_2}{2}\right)$ which leads to $\overline{\nabla g} = 0$ and therefore, $\dot{\hat{\alpha}} = 0$ when α is at the edge of $g(\alpha) \leqslant 0$.

In the sequel, we examine the performance of the proposed method on a fully parallel manipulator and a redundant cable driven robot. In order to test the efficiency of the method in experiment, ARAS suspended CDR is used and the proposed method with projection algorithm is implemented and compared to that of a simple controller on the calibrated model. Due to the fact that unmodeled dynamics are inevitable in experiment, a simulation on a 2-DOF R$\underline{\text{P}}$R robot is also considered and the proposed method is compared to that of an adaptive robust controller.

4. Simulation results

In this section, simulation results of the proposed method on a 2-DOF R$\underline{\text{P}}$R parallel robot is presented. The schematic of this robot is illustrated in Fig. 2, where $X = [x, y]^T$ denotes position of end-effector, I_{xi} denotes the moment of inertial of i-th link, m_{i1} and m_{i2} are the mass of i-th cylinder and piston, respectively, and m_p denotes the mass of the end-effector. The dynamic matrices and the Jacobian matrix of the robot may be given by

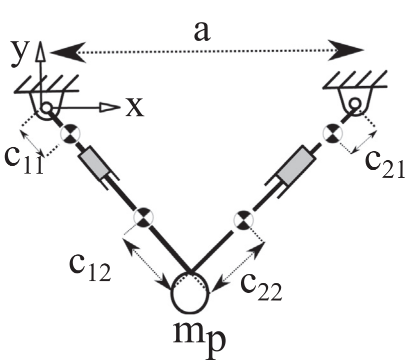


Fig. 2. Schematic of the 2-DOF R$\underline{\text{P}}$R parallel robot.

$$M = m_p I_2 + \sum_{i=1}^{2} m_{i2}\hat{\lambda}_i\hat{\lambda}_i^T - \tfrac{1}{l_i^2} I_{xi}\hat{\lambda}_{i\times}^2 - m_{ce}\hat{\lambda}_{i\times}^2,$$
$$C = \sum_{i=1}^{2} -\tfrac{2}{l_i} m_{co}\dot{l}_i\hat{\lambda}_{i\times}^2 - \tfrac{1}{l_i^2} m_{i2}c_{i2}\hat{\lambda}_i\dot{X}^T\hat{\lambda}_{i\times}^2,$$
$$G = \left(m_p + \sum_{i=1}^{2} m_{ge}\hat{\lambda}_{i\times}^2 - m_{i2}\hat{\lambda}_i\hat{\lambda}_i^T\right)\begin{bmatrix}0\\ g\end{bmatrix}, \qquad J = -\begin{bmatrix}\frac{x}{l_1} & \frac{y}{l_1}\\ \frac{x-a}{l_2} & \frac{y}{l_2}\end{bmatrix}, \quad \hat{\lambda}_1 = \begin{bmatrix}\frac{x}{l_1}\\ \frac{y}{l_1}\end{bmatrix}, \quad \hat{\lambda}_2 = \begin{bmatrix}\frac{x-a}{l_2}\\ \frac{y}{l_2}\end{bmatrix}$$

with

$$l_1^2 = x^2 + y^2, \qquad l_2^2 = (x-a)^2 + y^2, \qquad \dot{l}_i = J_{i,1}\dot{x} + J_{i,2}\dot{y}, \qquad m_{ce} = \sum_{i=1}^{2}\tfrac{1}{l_i^2}\left(m_{i1}c_{i1}^2 + m_{i2}c_{i2}^2\right)$$
$$m_{co} = \tfrac{1}{l_i} m_{i2}c_{i2} - \tfrac{1}{l_i^2}\left(I_{xi} + l_i^2 m_{ce}\right), \qquad m_{ge} = \tfrac{1}{l_i}\left(m_{i1}c_{i1} + m_{i2}(l_i - c_{i2})\right)$$
$$\hat{\lambda}_{1\times}^2 = \begin{bmatrix}-\frac{y^2}{l_1^2} & \frac{xy}{l_1^2}\\ \frac{xy}{l_1^2} & -\frac{x^2}{l_1^2}\end{bmatrix}, \qquad \hat{\lambda}_{2\times}^2 = \begin{bmatrix}-\frac{y^2}{l_2^2} & \frac{(x-a)y}{l_2^2}\\ \frac{(x-a)y}{l_2^2} & -\frac{(x-a)^2}{l_2^2}\end{bmatrix}$$

The parametric values of the robot considered in the simulations are given in Table 1. The mass of end-effector is considered equal to 2Kg. Note that Jacobian matrix can be rewritten as

$$J = -\begin{bmatrix}x & y\\ x-a & y\end{bmatrix}\begin{bmatrix}\frac{1}{l_1} & 0\\ 0 & \frac{1}{l_2}\end{bmatrix} = -J_{new}L^{-1}, \tag{27}$$

and then J_{new} can be expressed in regressor form (9) as follows

$$J_{new} = \begin{bmatrix}x & y\\ x & y\end{bmatrix} + \begin{bmatrix}0 & 0\\ -1 & 0\end{bmatrix}a = \begin{bmatrix}x & y\\ x & y\end{bmatrix} + Y\Theta.$$

The other regressors are presented in [2].

In order to evaluate performance of proposed method in Theorem 1, a simulation with adaptive robust controller proposed in [24] is also considered. In order to test the performance of the both parts of controller, i.e. adaptive and robust part, a bounded external disturbance which satisfies $\|d\| < 1$ is also considered. The parameters of the robot are perturbed 25% from its nominal values. The gains of controller are chosen as

$$\Gamma = 2.5I, \qquad K = 4I, \qquad K' = I, \qquad \Lambda = 5I.$$

The gains are chosen such that convergence rate is moderate, configuration variables suitably converges to their desired values and control law is smooth and without oscillations. Simulation results are illustrated in Fig. 3. As it is seen in this figure, the configuration variables of the robot converge to the desired values in both methods. However, the control signal corresponding to the adaptive robust controller has an undesirable chattering which is not practically implementable. Note that as indicated in [24], it is possible to avoid chattering with the expense of reducing the asymptotic stability by UUB stability. However, in the proposed method asymptotic stability is achievable with smoother control efforts.

5. Experimental results

In order to verify the performance of the proposed method in practice, a 3–DOF redundant cable driven robot is considered. As it is shown in the schematic of the robot in Fig. 4, The end-effector is suspended from anchor points by four cables. All of the anchor points are installed at the same height. The robot has three translational degrees of freedom with four actuated cables which are driven by motors through pulleys. Kinematics formulation of this robot is given by

$$l_i^2 = (x - x_{Ai})^2 + (y - y_{Ai})^2 + (z - z_{Ai})^2 \quad i = 1, \ldots, 4 \tag{28}$$

where, $X = [x, y, z]^T$ is the position of end-effector and x_{Ai}, y_{Ai}, z_{Ai} are the uncertain kinematic parameters that determine the cable anchor points. Dynamic matrices of the robot with the assumption of massless and infinitely stiff cables are as follows

Table 1
Parametric values of 2R$\underline{P}$R robot.

	m_{i1}	m_{i2}	c_{i1}	c_{i2}	I_{xi}
$i = 1$	1	1	0.5	0.5	0.1
$i = 2$	1	1	0.5	0.5	0.1

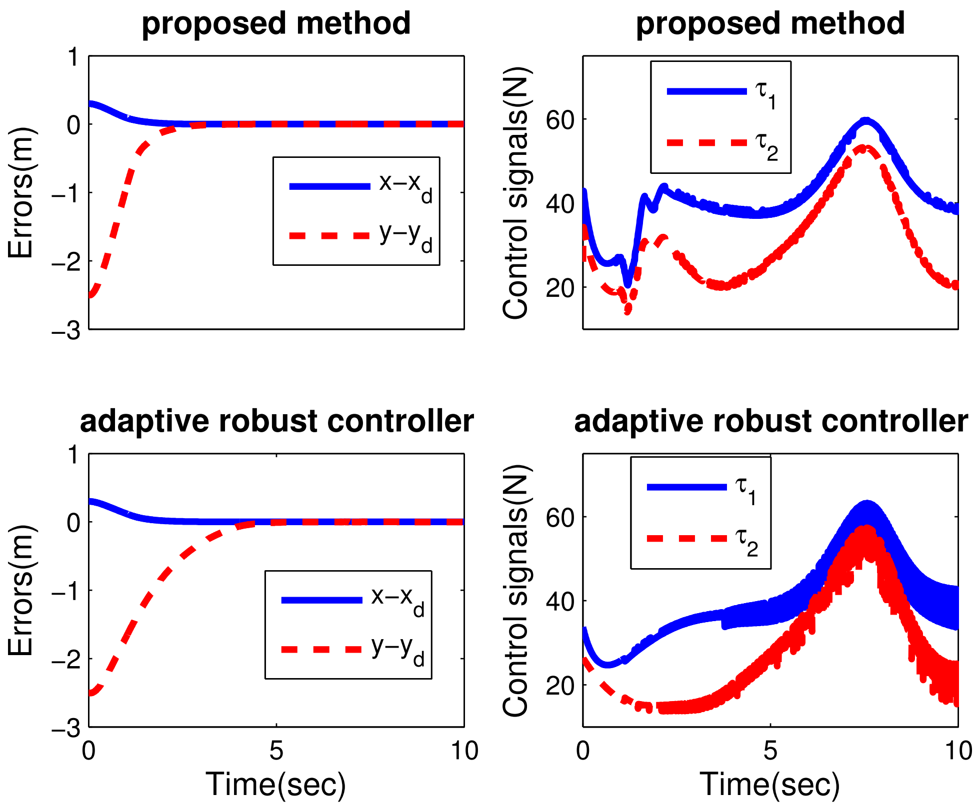


Fig. 3. Tracking error converges to zero with both method while chattering in control law is destructive inevitable part of adaptive robust controller.

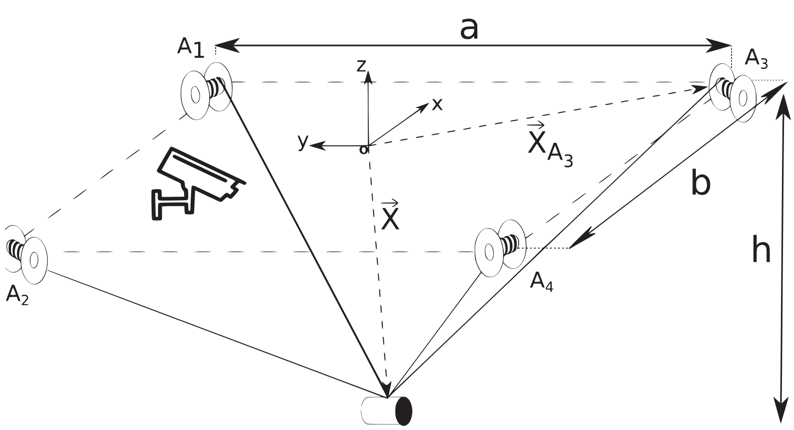


Fig. 4. Schematic of the suspended robot with four cables. All of the anchor points are at the same height. Center of coordinates is located in the middle of $A_1 - A_2 - A_3 - A_4$ rectangle with zero height.

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}, \qquad C = 0_{3\times3}, \qquad G = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{29}$$

where m is the mass of end-effector.

Since the proposed method is also applicable to redundantly actuated parallel robots, the experiment is designed such that the method is applied to a redundant CDR. The Jacobian matrix may be rearranged into the following form:

$$J^T = -\begin{bmatrix} x - x_{A1} & x - x_{A2} & x - x_{A3} & x - x_{A4} \\ y - y_{A1} & y - y_{A2} & y - y_{A3} & y - y_{A4} \\ z - z_{A1} & z - z_{A2} & z - z_{A3} & z - z_{A4} \end{bmatrix} \begin{bmatrix} \frac{1}{l_1} & 0 & 0 & 0 \\ 0 & \frac{1}{l_2} & 0 & 0 \\ 0 & 0 & \frac{1}{l_3} & 0 \\ 0 & 0 & 0 & \frac{1}{l_4} \end{bmatrix}. \tag{30}$$

Thus, Jacobian matrix is expressed in the form $J^T = J^T_{new}(X)L^{-1}$. Now it is possible to express J^T_{new} in regressor form

$$J^T_{new} = \begin{bmatrix} x & x & x & x \\ y & y & y & y \\ z & z & z & z \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_{A1} & x_{A2} & x_{A3} & x_{A4} \\ y_{A1} & y_{A2} & y_{A3} & y_{A4} \\ z_{A1} & z_{A2} & z_{A3} & z_{A4} \end{bmatrix} = \begin{bmatrix} x & x & x & x \\ y & y & y & y \\ z & z & z & z \end{bmatrix} + Y\Theta. \tag{31}$$

Note that J_{new} does not satisfy the assumption in [10,11], i.e. $\widehat{J}^{\dagger}_{new}\xi$ with $\xi \in \mathbb{R}^3$ as a known vector, may not been expressed in regressor form due to fractional elements generated by determinant. Other regressors are presented in [2].

In order to measure the length of cables, the motor rotation angles are measured by incremental encoders. Hence, the current length of cables are available by knowing initial length of the cables. A 100 frame per second stereo vision camera with 640×480 resolution is utilized to measure position of the LED lamp as the position of the end-effector. More information about the experimental setup was given in [19]. Fig. 5 shows different parts of ARAS suspended cable driven robot.

The mass of end-effector is equal to 4.5 KG and coordinates of cable anchor points are obtained by calibration as:

$$\begin{aligned} x_{A1} &= -x_{A2} = x_{A3} = -x_{A4} = \tfrac{b}{2} = \tfrac{3.56}{2} \\ y_{A1} &= y_{A2} = -y_{A3} = -y_{A4} = \tfrac{a}{2} = \tfrac{7.05}{2} \\ z_{A1} &= z_{A2} = z_{A3} = z_{A4} = h = 4.26 \end{aligned} \tag{32}$$

The spring-like desired trajectory is expressed in SI unit systems, as follows:

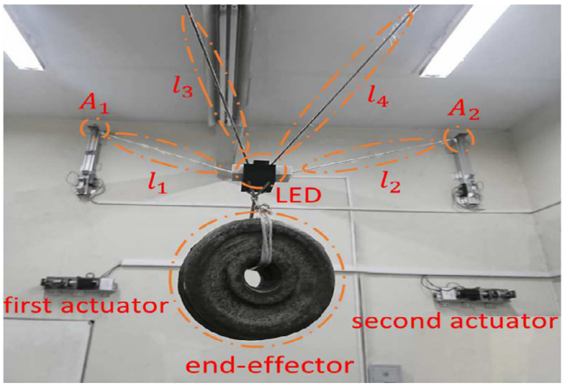
$$\begin{cases} x_d(t) = 0.48 - 0.1\cos\left(\frac{2\pi}{5}t\right) \\ y_d(t) = -0.22 + 0.1\sin\left(\frac{2\pi}{5}t\right) \\ z_d(t) = 1.5 + 0.0075t \end{cases} \tag{33}$$

The center and diameter of the trajectory are chosen in such a way that the robot is inside its workspace away from its singular space, and well-measured by the stereo camera. The gains of controller are set to:

$$\Gamma = 20I, \qquad K = 10I, \qquad \Lambda = 5I,$$

and K' is considered equal to zero to test merely the performance of adaptive part of controller. The gains are chosen by some trials and errors such that a suitable performance is achieved. The initial position of the robot is $[x_0, y_0, z_0]^T = [0.43, -0.28, 1.5]^T$. Notice that in contrast to all previous works on ARAS CDR, in this work the initial position of the robot is not on the trajectory, i.e. $\widetilde{X}$ is not zero at $t = 0$. Note that such sudden motions request in CDRs may lead to longitudinal and transverse oscillations in cables which may cause instabilities in the robot. This extreme scenario is tested on the robot with suitable controller performance.

The upper bound of perturbation for dynamic and kinematic parameters is set to 10%. In order to examine the effect of the projection algorithm, a saturation function is used as a simple appropriate projection. By this means, estimated param-



(a) Various components of ARAS CDR

(b) The prototype of a stereo vision system, attached to the ceiling of the lab at the geometrical center of the workspace.

Fig. 5. ARAS suspended cable driven robot.

eters are saturated within the $\pm 15\%$ bounds. For the sake of comparison, and in order to analyze the performance of the proposed method, a non-adaptive controller is also implemented on to the robot. The implemented control law is as what given in (12) with the parameters obtained by kinematic and dynamic calibration. By this means a well suited control law is considered, while the exhaustive procedure of calibration is needed. Note that a high gain controller was also implemented on the robot, whose results are not reported in this paper, since it led to high oscillations in the cables.

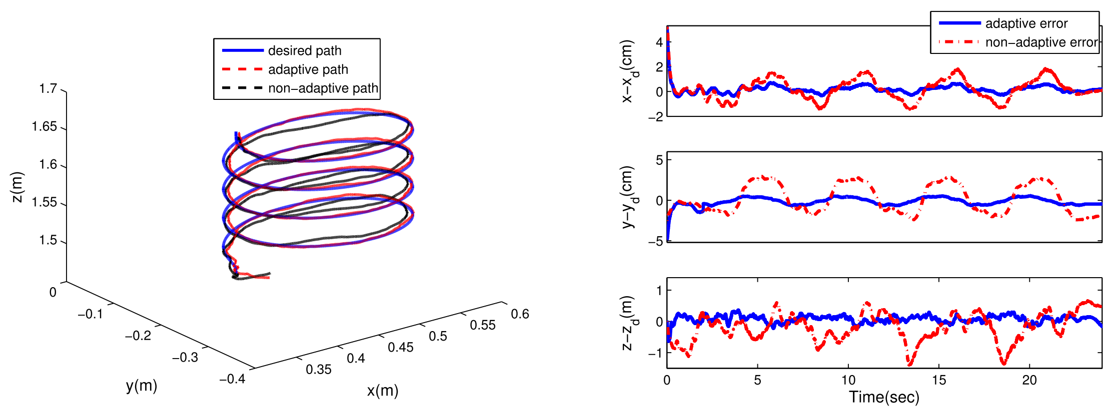
The experimental results are illustrated in Fig. 6 and Fig. 7. As it is seen in Fig. 6(a), the traversed path with adaptive controller suitably tracks the desired path with a short transient. However, non-adaptive controller is not that precise, and leads to larger tracking error within path. In order to compare the results more clearly, the tracking errors are shown in Fig. 6(b). Note that the tracking error illustrated in these figures are in centimeters. The results show the desirable performance of the proposed method in comparison to non-adaptive controller based on the calibrated model. The response of the system is affected by the oscillations of cables at initial times due to an initial error between the trajectory and position of the robot. After this period, fluctuations are suitably damped and thus, the robot has a suitable performance.

In Fig. 6(b), the tracking error in X direction is less than 0.5cm with adaptive controller while with the non-adaptive controller, it is about 2 cm. The tracking errors in Y and Z directions are about 0.5 cm and 0.25 cm for proposed method and 3 cm and 1.5 cm with the non-adaptive controller, respectively. This shows superiority of the proposed method compared to that of the sate-of-the-art controllers, in terms of tracking errors. Notice that the reason why error in Y direction is almost double of that in X direction, is the distance between anchor points proposed in (32). Recall that in the case of non-adaptive controller, the kinematic and dynamic parameters shall be obtained by an exhaustive calibration procedure. Indeed, if the parameters are not found precisely through such calibration procedure, and just the nominal values are used in the implementation, much worse tracking performance and instability may occur.

Fig. 7 shows control efforts for adaptive and non-adaptive controllers in experiments. As it is seen in this figure, some oscillations are observed at the initial moments. The main reason for such oscillations are the oscillations caused in the cables, because of its elasticity, while the reason why control laws with proposed method have smaller oscillations is the adaptation law. Note that all control signals are positive, since as explained in Remark 2, the desired trajectory is within the feasible workspace of the robot as well as using the projection algorithm, it is ensured that adapted parameters can not exceed from a specified bound. Finally, as it is depicted in this figure, the control efforts needed in the proposed adaptive controller are almost similar to that of non-adaptive controller, despite their suitable tracking performance.

6. Conclusions and prospect researches

This paper has been focused on the design of an adaptive tracking controller for parallel robots with dynamic and kinematic uncertainties. A novel expression for the inverse of Jacobian matrix in regressor form was introduced, a method based on passivity was proposed and an adaptation law for unknown parameters was elicited. By this means, it is proved that the tracking error of the robot converges asymptotically to zero in the presence of kinematics and dynamic uncertainties. The performance of the controller was verified through simulation and experiment, and it has been shown that in comparison to the state-of-the-art method, the response is improved, while the effect of projection in singularity avoidance was highlighted. Since the research on the control of parallel robots in presence of kinematic and dynamic uncertainties is developing,



(a) Path of the robot with adaptive and non-adaptive controllers. In contrast to non-adaptive response which is based on calibrated model, the path with adaptive controller is almost matched with the desired path after a transient response.

(b) Tracking error of x, y and z in centimeter. Maximum remaining error with adaptive controller is apparently fewer which shows superiority of proposed method compared to the method based on calibration.

Fig. 6. Experiment results of the adaptive and non-adaptive controllers on a 3-DOF CDR. Tracking errors with proposed controller is smaller than the non-adaptive controller based on calibrated model.

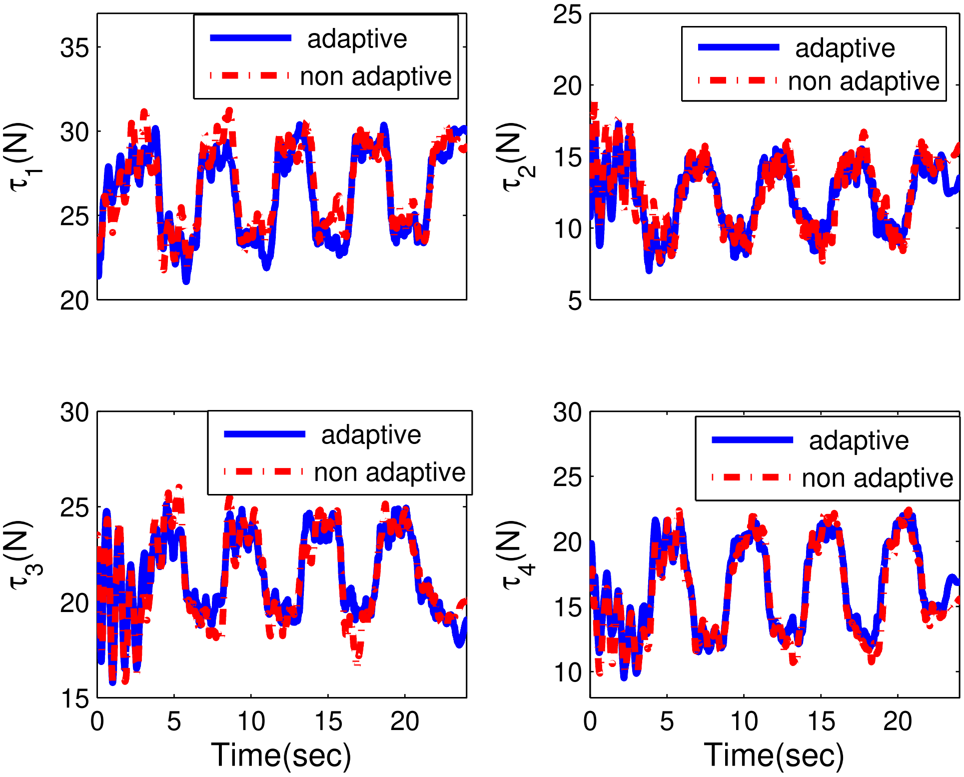


Fig. 7. Control efforts of adaptive and non-adaptive controllers in the experiment. The reason for oscillation at the initial moments are fluctuations in cables. The control laws with proposed method have more small fluctuations due to adaptation laws.

future research may be devoted to decoupling the adaptation laws for kinematic and dynamic parameters in order to reduce the number of adapting parameters. Extension of the proposed method to the case of serial robots is also underway.

CRediT authorship contribution statement

M. Reza J. Harandi: Conceptualization, Methodology, Software, Writing - original draft. **S.A. Khalilpour:** Software, Methodology, Data curation. **Hamid D. Taghirad:** Supervision, Methodology, Writing - review & editing. **Jose Guadalupe Romero:** Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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