

**Case study 1:** Consider the simple anthropomorphic arm in Fig.1. Several practical uses of this arm can be seen in [1](#) and [2](#).

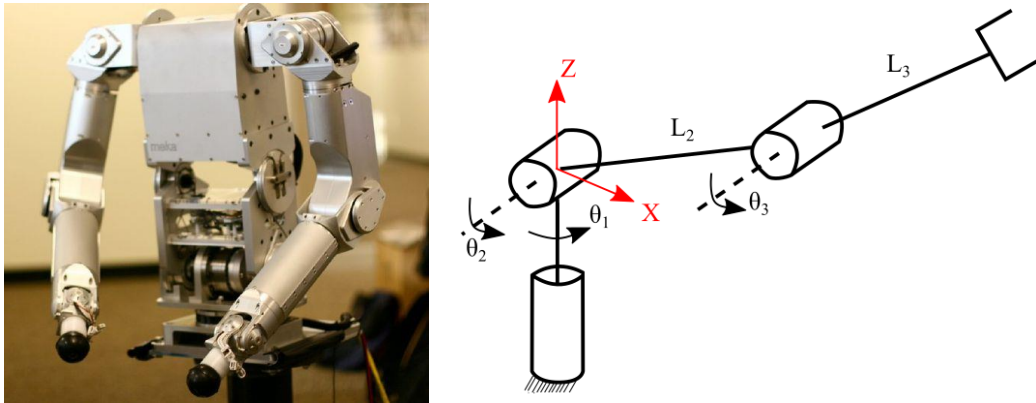


Fig1. Kinematic Structure of simple anthropomorphic arm

1. Plot isotropic configurations of the robot in a specific workspace, using manipulability index.
2. Plot singular configurations in a specific workspace, using manipulability index. Also, derive an analytical solution for singular configurations and verify your answer.

Use the following values for the robot:  $L_2 = 1$ ,  $L_3 = \sqrt{2}$ ,  $0^\circ \leq \theta_1, \theta_2, \theta_3 \leq 90^\circ$ ,  $\epsilon_{Threshold} = 0.01$

**Case study 2:** Palletizing is a demanding application of stacking boxes, bags, cases, bottles, and cartons onto pallets as the last step in the assembly line before being loaded onto a shipping truck. A schematic of a 3-DOF [palletizer robot](#) can be seen in Fig 2.

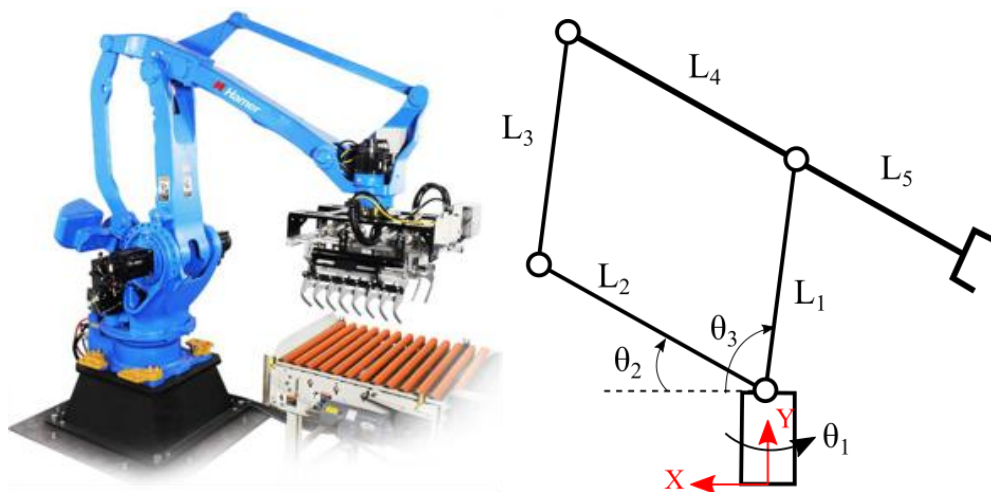


Fig2. Kinematic Structure of 3-DOF palletizer robot

1. Specify analytical conditions for  $\theta_3$  in the trajectory planning to avoid singular configurations.
2. Consider  $L_1 = 1$ ,  $L_5 = x \cdot L_1$ , and a specific trajectory. Derive an optimal value<sup>1</sup> of  $x$  to maximize global manipulability index<sup>2</sup>, to tend the robot to isotropic configurations. Use the following values for the robot:

$$0 < t < 1, 0 < x < 3$$

Considered Trajectory:  $\theta_1 = \frac{\pi}{2} \sin\left(\frac{\pi}{2}t\right)$ ,  $\theta_2 = \frac{\pi}{3} \sin\left(\frac{\pi}{2}t\right) + \frac{\pi}{2}$ ,  $\theta_3 = \frac{\pi}{6} \sin\left(\frac{\pi}{2}t\right) + \frac{\pi}{6}$

**Project:** Consider 3-DOF parallelogram-based robot in Fig.1. Such kinematic structure has been used in many surgical robots such as [Da Vinci](#) and [PRECEYES](#).

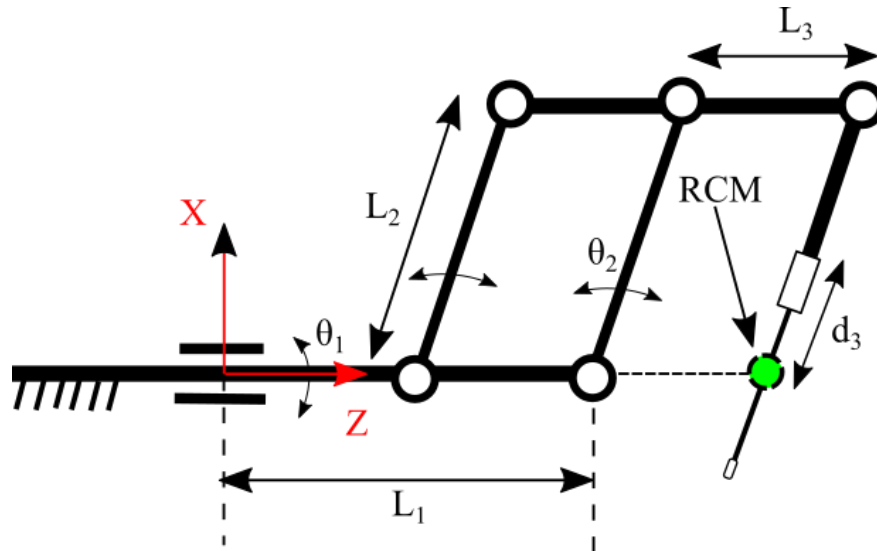


Fig1. Kinematic Structure of 3-DOF parallelogram based robot. To better understand the above schematic, see this [link](#).

1. Use Rcond kinematic index and plot singular configurations of the robot in a specific, reachable workspace.
2. Derive analytical solution for singular configurations and verify your answer.
3. Mention why the existing singularities are helpful in the context of a surgical robot's [application](#).

Use the following values for the robot:  $-45^\circ \leq \theta_1 \leq +45^\circ$ ,  $45^\circ \leq \theta_2 \leq +135^\circ$ ,  $0 \leq d_3 \leq 3cm$

## Appendix

Here are the Jacobian matrices to solve each question so that everyone gets the same answer.

### Case study 1:

<sup>1</sup> For example, use [fmincon](#) or [GA](#)

<sup>2</sup> Consider the objective function as:  $\ln\left(\int_0^1 \mu(t) dt - 1\right)$ , and use [numerical integration](#).

$$J := \begin{bmatrix} 0 & \sin(\theta_1) & \sin(\theta_1) \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) \\ 1 & 0 & 0 \\ -L_3 \sin(\theta_1) \cos(\theta_2 + \theta_3) - \sin(\theta_1) L_2 \cos(\theta_2) & -(L_3 \sin(\theta_2 + \theta_3) + L_2 \sin(\theta_2)) \cos(\theta_1) & -L_3 \cos(\theta_1) \sin(\theta_2 + \theta_3) \\ \cos(\theta_1) L_2 \cos(\theta_2) + L_3 \cos(\theta_1) \cos(\theta_2 + \theta_3) & -(L_3 \sin(\theta_2 + \theta_3) + L_2 \sin(\theta_2)) \sin(\theta_1) & -L_3 \sin(\theta_1) \sin(\theta_2 + \theta_3) \\ 0 & L_3 \cos(\theta_2 + \theta_3) + L_2 \cos(\theta_2) & L_3 \cos(\theta_2 + \theta_3) \end{bmatrix}$$

## Case study 2:

$$J := \begin{bmatrix} 0 & \sin(\theta_1) & 0 \\ 1 & 0 & 0 \\ 0 & \cos(\theta_1) & 0 \\ -\sin(\theta_1) (-L_5 \cos(\theta_2) + L_1 \cos(\theta_3)) & L_5 \sin(\theta_2) \cos(\theta_1) & -L_1 \sin(\theta_3) \cos(\theta_1) \\ 0 & -L_5 \cos(\theta_2) & L_1 \cos(\theta_3) \\ -\cos(\theta_1) (-L_5 \cos(\theta_2) + L_1 \cos(\theta_3)) & -L_5 \sin(\theta_1) \sin(\theta_2) & L_1 \sin(\theta_1) \sin(\theta_3) \end{bmatrix}$$

## Project:

$$J := \begin{bmatrix} 0 & -\sin(\theta_1) & 0 \\ 0 & \cos(\theta_1) & 0 \\ 1 & 0 & 0 \\ -\sin(\theta_1) \sin(\theta_2) d_3 & \cos(\theta_1) \cos(\theta_2) d_3 & \cos(\theta_1) \sin(\theta_2) \\ \cos(\theta_1) \sin(\theta_2) d_3 & \sin(\theta_1) \cos(\theta_2) d_3 & \sin(\theta_1) \sin(\theta_2) \\ 0 & -\sin(\theta_2) d_3 & \cos(\theta_2) \end{bmatrix}$$

**Code language:** It's recommended to use MATLAB to develop your codes. But you can use other symbolic software like Maple as well.

**How to contact teaching assistants:** If you have any questions about these exercises, please contact us ONLY through Email.

**How to submit:** Zip your files (codes, reports, etc.) within the format of HW#\_Name\_StudentID, and submit them to the LMS website.

**Late homework policy:** Please submit your assignments on time, in case you run out of time; you can still send it as late submission only by one day with the expense of losing some marks depending on the time you submit your work.

**Collaboration policy:** Collaboration with humans is very beneficial but restricted to the "whiteboard level," meaning that we recommend you to discuss approaches and solutions with your peers, but write your code, reports, and analytical derivations by yourself.

Good Luck!