به نام آنکه جان را فکرت آموخت

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رباتیک

پاسخ تمرین سری سوم

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دانشگاه صنعتی خواجه نصیرالدین طوسی
استاد: حمیدرضا تقی راد

Investigate the Maple codes first, then read this report.

Case Study1:

For the anthropomorphic arm, corresponding D-H parameters are as Table 1:

$Joint_i$	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	0	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3

Table 1 D-H parameters of anthropomorphic arm

Transformation Matrices (DH parameters):

$$T_{12} = T_1 T_2$$

$$T_{End-Effector} = T_{123} = T_1 T_2 T_3$$

Rotation Matrices:

$$R_1 = T_1(1..3,1 \dots 3)$$

$$R_2 = T_{12}(1..3,1 \dots 3)$$

$$R_3 = T_{123}(1..3,1 \dots 3)$$

So, we have:

$$R_1 := \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_2 := \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & \sin(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & -\cos(\theta_1) \\ \sin(\theta_2) & \cos(\theta_2) & 0 \end{bmatrix}$$

$$R_3 := \begin{bmatrix} \cos(\theta_1)\cos(\theta_2+\theta_3) & -\cos(\theta_1)\sin(\theta_2+\theta_3) & \sin(\theta_1) \\ \sin(\theta_1)\cos(\theta_2+\theta_3) & -\sin(\theta_1)\sin(\theta_2+\theta_3) & -\cos(\theta_1) \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 \end{bmatrix}$$

Therefore, we may derive screw based jacobian, as follow:

$\boldsymbol{i} = \mathbf{3}, s_4 = [0,0,1]^T, s_{o_4} = [0,0,0]^T$

$\boldsymbol{i} = \mathbf{2}, s_3 = R_2[0,0,1]^T, s_{o_3} = s_{o_4} - R_3[L_3, 0,0]^T$

$\boldsymbol{i} = \mathbf{1}, s_2 = R_1[0,0,1]^T, s_{o_2} = s_{o_3} - R_2[L_2, 0,0]^T$

$\boldsymbol{i} = \mathbf{0}, s_1 = [0,0,1]^T, s_{o_1} = s_{o_2} - R_1[0,0,0]^T$

Finally:

$J_1 = [s_1, s_{o_1} \times s_1]$, $J_2 = [s_2, s_{o_2} \times s_2], J_3 = [s_3, s_{o_3} \times s_3]$

$J = [J_1, J_2, J_3]$

Final result:

$$J := \begin{bmatrix} 0 & \sin(\theta_1) & \sin(\theta_1) \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) \\ 1 & 0 & 0 \\ -L_3\cos(\theta_2+\theta_3)\sin(\theta_1) - \sin(\theta_1)L_2\cos(\theta_2) & -(L_3\sin(\theta_2+\theta_3) + L_2\sin(\theta_2))\cos(\theta_1) & -L_3\cos(\theta_1)\sin(\theta_2+\theta_3) \\ \cos(\theta_1)L_2\cos(\theta_2) + L_3\cos(\theta_1)\cos(\theta_2+\theta_3) & -(L_3\sin(\theta_2+\theta_3) + L_2\sin(\theta_2))\sin(\theta_1) & -L_3\sin(\theta_1)\sin(\theta_2+\theta_3) \\ 0 & L_3\cos(\theta_2+\theta_3) + L_2\cos(\theta_2) & L_3\cos(\theta_2+\theta_3) \end{bmatrix}$$

Second Method:

$J_v = Jacobian([X,Y,Z],[\theta_1,\theta_2,\theta_3])$, use Jacobian command in MATLAB or Maple.

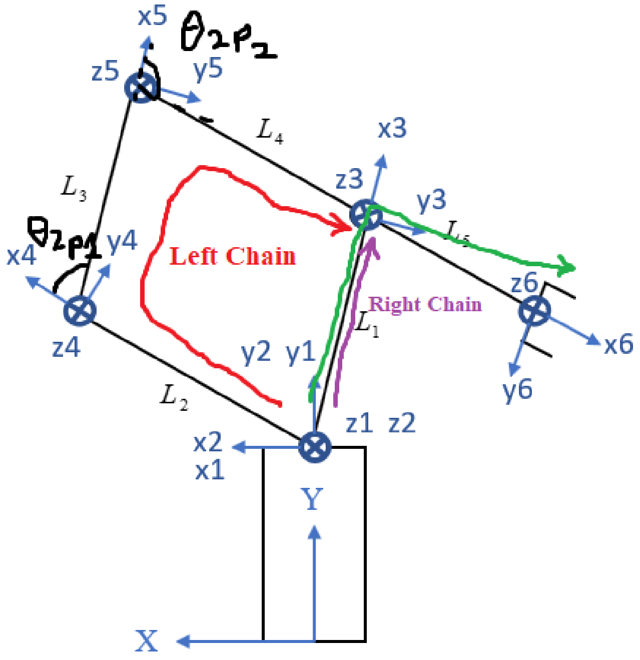
$J_w = [z_1, z_2, z_3]$

Where $z_1 = R_1(1\ldots3,3)$, $z_2 = R_2(1\ldots3,3)$, $z_3 = R_3(1\ldots3,3)$.

Finally:

$$J = \begin{bmatrix} J_w \\ J_v \end{bmatrix}$$

Case Study 2.



$Joint_i$	α_i	a_i	d_i	θ_i
1_R	$-\pi/2$	0	0	0
2_R	$\pi/2$	0	0	θ_1
3_R	0	L_1	0	θ_3

Table 2 D-H parameters of right loop of parallelogram arm

$Joint_i$	α_i	a_i	d_i	θ_i
1_L	$-\pi/2$	0	0	0
2_L	$\pi/2$	0	0	θ_1
3_L	0	L_2	0	θ_{2p1}
4_L	0	L_3	0	θ_{2p2}
5_L	0	L_4	0	0

Table 3 D-H parameters of left loop of parallelogram arm

Left and right chain equality condition:

$\theta_{2p1} = \theta_3 - \theta_2$

$\theta_{2p2} = \pi - \theta_{2p1}$

$Joint_i$	α_i	a_i	d_i	θ_i
1	$-\pi/2$	0	0	0
2	$\pi/2$	0	0	θ_1
3	0	L_1	0	θ_3
4	0	L_5	0	θ_{2p2}

Table 4 D-H parameters of end-effector

Method 1: Left Loop

With proper kinematic analysis of left loop:

$$R_1 := \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & \sin(\theta_1) \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ -\sin(\theta_1)\cos(\theta_2) & \sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) \end{bmatrix}$$

$$R_2 := \begin{bmatrix} \cos(\theta_1)\cos(\theta_3) & -\cos(\theta_1)\sin(\theta_3) & \sin(\theta_1) \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ -\sin(\theta_1)\cos(\theta_3) & \sin(\theta_1)\sin(\theta_3) & \cos(\theta_1) \end{bmatrix}:$$

$$R_3 := \begin{bmatrix} -\cos(\theta_1)\cos(\theta_2) & \cos(\theta_1)\sin(\theta_2) & \sin(\theta_1) \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) \end{bmatrix}$$

$$\dot{q}_{RL} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 - \dot{\theta}_2, -(\dot{\theta}_3 - \dot{\theta}_2)]^T$$

$$\dot{q} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3\,]^T$$

1. Angular part:

$J_{w_1} = z_0 =$ **$[0, 1, 0]^T$, according to base coordinate systesm**

$J_{w_2} = z_1 = R_1(1 \ldots 3, 1 \ldots 3)$

$J_{w_3} = z_2 = R_2(1 \ldots 3, 1 \ldots 3)$

$J_{w_4} = z_3 = R_3(1 \dots 3, 1 \dots 3)$

$J_{w_{RL}} = [J_{w_1}, J_{w_2}, J_{w_3}, J_{w_4}]$

$[J_{w_1}, J_{w_2}, J_{w_3}, J_{w_4}]\boldsymbol{[\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 - \dot{\theta}_2, -(\dot{\theta}_3 - \dot{\theta}_2)]^T = [J_{w_1}, J_{w_2} - J_{w_3} + J_{w_4}, J_{w_3} + J_{w_4}][\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T}$

So, final jacobian will be derived as:

$$J_w = \left[J_{w_1}, J_{w_2} - J_{w_3} + J_{w_4}, J_{w_3} + J_{w_4}\right]$$

2. Linear part:

$P_{44} = [0,0,0]^T$

$P_{34} = R_3[L_5 + L_4, 0,0]^T + P_{44}$

$P_{24} = R_2[L_3, 0,0]^T + P_{34}$

$P_{14} = R_1[L_2, 0,0]^T + P_{24}$

$P_{04} = [0,0,0]^T + P_{14}$

$J_{v_1} = [z_0 \times P_{04}]$, $J_{v_2} = [z_1 \times P_{14}]$, $J_{v_3} = [z_2 \times P_{24}]$, $J_{v_4} = [z_3 \times P_{34}]$

$J_{v_{RL}} = [J_{v_1}, J_{v_2}, J_{v_3}, J_{v_4}]$

$[J_{v_1}, J_{v_2}, J_{v_3}, J_{v_4}]\boldsymbol{[\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 - \dot{\theta}_2, -(\dot{\theta}_3 - \dot{\theta}_2)]^T = [J_{v_1}, J_{v_2} - J_{v_3} + J_{v_4}, J_{v_3} + J_{v_4}][\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T}$

So, final jacobian will be derived as:

$$J_v = \left[J_{v_1}, J_{v_2} - J_{v_3} + J_{v_4}, J_{v_3} + J_{v_4}\right]$$

Final Jacobian:

$$J = \begin{bmatrix} J_w \\ J_v \end{bmatrix}$$

$$J := \begin{bmatrix} 0 & \sin(\theta_1) & 0 \\ 1 & 0 & 0 \\ 0 & \cos(\theta_1) & 0 \\ -\sin(\theta_1)\,(-L_5\cos(\theta_2)+L_1\cos(\theta_3)) & L_5\sin(\theta_2)\cos(\theta_1) & -L_1\sin(\theta_3)\cos(\theta_1) \\ 0 & -L_5\cos(\theta_2) & L_1\cos(\theta_3) \\ -\cos(\theta_1)\,(-L_5\cos(\theta_2)+L_1\cos(\theta_3)) & -L_5\sin(\theta_1)\sin(\theta_2) & L_1\sin(\theta_1)\sin(\theta_3) \end{bmatrix}$$

Method 2: Right Loop

$$\dot{q}_{LL} = [\dot{\theta}_1, \dot{\theta}_3, -(\dot{\theta}_3 - \dot{\theta}_2)]^T$$

$$\dot{q} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3\,]^T$$

$$R1 := \begin{bmatrix} \cos(\theta_1)\cos(\theta_3) & -\cos(\theta_1)\sin(\theta_3) & \sin(\theta_1) \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ -\sin(\theta_1)\cos(\theta_3) & \sin(\theta_1)\sin(\theta_3) & \cos(\theta_1) \end{bmatrix}$$

$$R2 := \begin{bmatrix} -\cos(\theta_1)\cos(\theta_2) & \cos(\theta_1)\sin(\theta_2) & \sin(\theta_1) \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) \end{bmatrix}$$

1. Angular part:

$J_{w_1} = z_0 = [0, 1, 0]^T$, **according to base coordinate systesm**

$$J_{w_2} = z_1 = R_1(1 \dots 3, 1 \dots 3)$$

$$J_{w_3} = z_2 = R_2(1 \dots 3, 1 \dots 3)$$

$$J_{w_{LL}} = [J_{w_1}, J_{w_2}, J_{w_3}]$$

$$[J_{w_1}, J_{w_2}, J_{w_3}][\dot{\theta}_1, \dot{\theta}_3, -(\dot{\theta}_3 - \dot{\theta}_2)]^T = [J_{w_1}, J_{w_3}, J_{w_2} - J_{w_3}\,][\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3\,]^T$$

So, final jacobian will be derived as:

$$J_w = \left[J_{w_1}, J_{w_3}, J_{w_2} - J_{w_3}\,\right]$$

2. Linear part:

$P_{33} = [0,0,0]^T$

$P_{23} = R_2[L_5, 0,0]^T + P_{33}$

$P_{13} = R_1[L_1, 0,0]^T + P_{23}$

$P_{03} = 0[0,0,0]^T + P_{13}$

$J_{v_1} = [z_0 \times P_{03}]$, $J_{v_2} = [z_1 \times P_{13}]$, $J_{v_3} = [z_2 \times P_{23}]$

$J_{v_{LL}} = [J_{v_1}, J_{v_2}, J_{v_3}]$

$[J_{v_1}, J_{v_2}, J_{v_3}][\dot{\boldsymbol{\theta}}_1, \dot{\boldsymbol{\theta}}_3, -(\dot{\boldsymbol{\theta}}_3 - \dot{\boldsymbol{\theta}}_2)]^T = [\boldsymbol{J}_{v_1}, \boldsymbol{J}_{v_3}, \boldsymbol{J}_{v_2} - \boldsymbol{J}_{v_3}][\dot{\boldsymbol{\theta}}_1, \dot{\boldsymbol{\theta}}_2, \dot{\boldsymbol{\theta}}_3]^T$

So, final jacobian will be derived as:

$J_v = \left[J_{v_1}, J_{v_3}, J_{v_2} - J_{v_3}\right]$

Final Jacobian:

$J = \begin{bmatrix} J_w \\ J_v \end{bmatrix}$

$$J := \begin{bmatrix} 0 & \sin(\theta_1) & 0 \\ 1 & 0 & 0 \\ 0 & \cos(\theta_1) & 0 \\ -\sin(\theta_1)\left(-L_5\cos(\theta_2) + L_1\cos(\theta_3)\right) & L_5\sin(\theta_2)\cos(\theta_1) & -L_1\sin(\theta_3)\cos(\theta_1) \\ 0 & -L_5\cos(\theta_2) & L_1\cos(\theta_3) \\ -\cos(\theta_1)\left(-L_5\cos(\theta_2) + L_1\cos(\theta_3)\right) & -L_5\sin(\theta_1)\sin(\theta_2) & L_1\sin(\theta_1)\sin(\theta_3) \end{bmatrix}$$

Project:

$$R_1 := \begin{bmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) \\ 0 & -1 & 0 \end{bmatrix}$$

$$R_2 = R_3 = \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & -\sin(\theta_1) & \cos(\theta_1)\sin(\theta_2) \\ \sin(\theta_1)\cos(\theta_2) & \cos(\theta_1) & \sin(\theta_1)\sin(\theta_2) \\ -\sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix}$$

- Screw based Jacobian:

$\boldsymbol{i} = \mathbf{3}, s_4 = [0,0,1]^T, s_{O_4} = [0,0,0]^T$

$\boldsymbol{i} = \mathbf{2}, s_3 = R_2[0,0,1]^T, s_{O_3} = s_{O_4} - R_3[0,0,d_3]^T$

$\boldsymbol{i} = \mathbf{1}, s_2 = R_1[0,0,1]^T, s_{O_2} = s_{O_3} - R_2[0,0,0]^T$

$\boldsymbol{i} = \mathbf{0}, s_1 = [0,0,1]^T, s_{O_1} = s_{O_2} - R_1[0, L_1 + L_3, 0]^T$

Finally:

$J_1 = [s_1, s_{O_1} \times s_1]$, $J_2 = [s_2, s_{O_2} \times s_2]$, $J_3 = [[0,0,0]^T, s_3]$

$J = [J_1, J_2, J_3]$

- DH Jacobian

$J_{w_1} = z_0 = [0,0,1]^T$

$J_{w_2} = z_1 = R_1(1 \dots 3, 1 \dots 3)$

$J_{w_3} = [0,0,0]^T$

$J_w = [J_{w_1}, J_{w_2}, J_{w_3}]$

$P_{33} = [0,0,0]^T$

$P_{23} = R_2[0,0,d_3]^T + P_{33}$

$P_{13} = R_1[0,0,0]^T + P_{23}$

$P_{03} = [0,0,L_1 + L_3]^T + P_{13}$

$J_{v_1} = [z_0 \times P_{03}]$, $J_{v_2} = [z_1 \times P_{13}]$, $J_{v_3} = [z_2]$

$J_v = [J_{v_1}, J_{v_2}, J_{v_3}]$

Finally:

$$J = \begin{bmatrix} J_w \\ J_v \end{bmatrix}$$

- Analytical Method:

$J_v = Jacobian([X, Y, Z], [\theta_1, \theta_2, d_3])$, use Jacobian command in MATLAB or Maple.

Finally:

$$J := \begin{bmatrix} 0 & -\sin(\theta_1) & 0 \\ 0 & \cos(\theta_1) & 0 \\ 1 & 0 & 0 \\ -\sin(\theta_1)\sin(\theta_2)\, d_3 & \cos(\theta_1)\cos(\theta_2)\, d_3 & \cos(\theta_1)\sin(\theta_2) \\ \cos(\theta_1)\sin(\theta_2)\, d_3 & \sin(\theta_1)\cos(\theta_2)\, d_3 & \sin(\theta_1)\sin(\theta_2) \\ 0 & -\sin(\theta_2)\, d_3 & \cos(\theta_2) \end{bmatrix}$$